

# Multi-Dimensional Risk and Performance Analysis for Equity Portfolios

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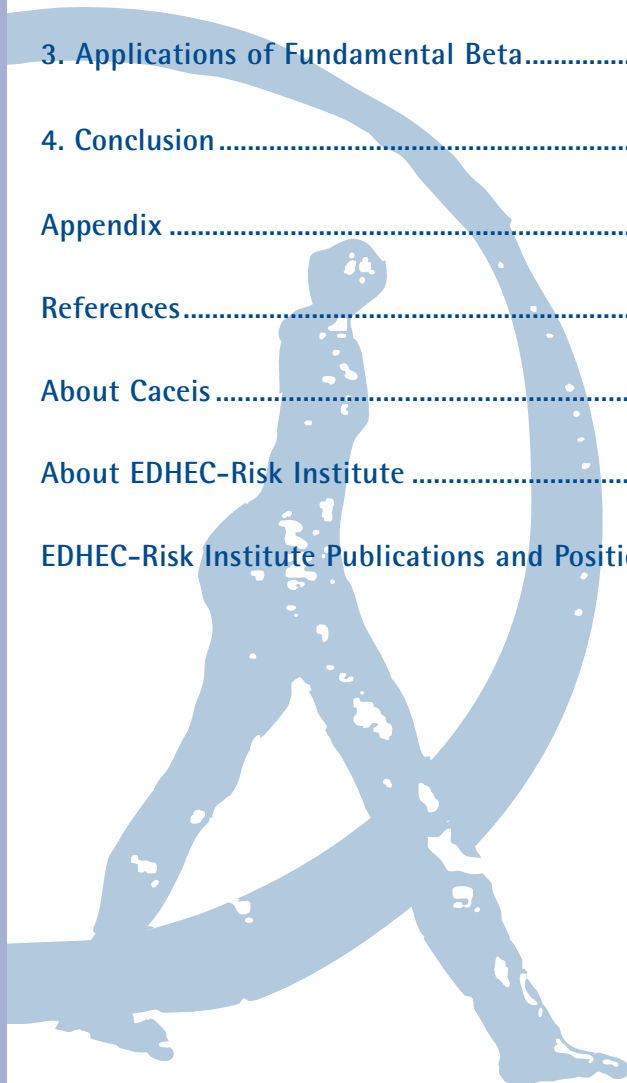


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# Foreword

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The present publication is drawn from the CACEIS research chair on "New Frontiers in Risk Assessment and Performance Reporting" at EDHEC-Risk Institute. This chair looks at improved risk reporting, integrating the shift from asset allocation to factor allocation, improved geographic segmentation for equity investing, and improved risk measurement for diversified equity portfolios.

Multi-factor models are standard tools for analysing the performance and the risk of equity portfolios. In addition to analysing the impact of common factors, equity portfolio managers are also interested in analysing the role of stock-specific attributes in explaining differences in risk and performance across assets and portfolios.

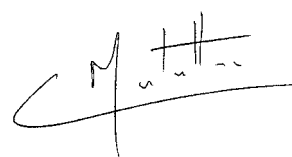
In this study, EDHEC-Risk Institute explores a novel approach to address the challenge raised by the standard investment practice of treating attributes as factors, with respect to how to perform a consistent risk and performance analysis for equity portfolios across multiple dimensions that incorporate micro attributes.

EDHEC-Risk Institute's study suggests a new dynamic meaningful approach, which consists in treating attributes of stocks as instrumental variables to estimate betas with respect to risk factors for explaining notably the cross-section of expected returns. In one example of implementation, the authors maintain a limited number of risk factors by considering a one-factor model, and they estimate a conditional beta that depends on the same three characteristics that define the Fama-French and Carhart factors.

In so doing, the authors introduce an alternative estimator for the conditional beta, which they name "fundamental beta" (as opposed to historical beta) because it is defined as a function of the stock's characteristics, and they provide evidence of the usefulness of these fundamental betas for (i) parsimoniously embedding the sector dimension in multi-factor portfolio risk and performance analysis, (ii) building equity portfolios with controlled target factor exposure, and also (iii) explaining the cross-section of expected returns, by showing that a conditional CAPM based on this "fundamental" beta can capture the size, value and momentum effects as well as the Carhart model, but without the help of additional factors.

I would like to thank my co-authors Kevin Giron and Vincent Milhau for their useful work on this research, and Laurent Ringelstein and Dami Coker for their efforts in producing the final publication. We would also like to extend our warmest thanks to our partners at CACEIS for their insights into the issues discussed and their commitment to the research chair.

We wish you a useful and informative read.



Lionel Martellini  
Professor of Finance,  
Director of EDHEC-Risk Institute

# About the Authors

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# Executive Summary



# Executive Summary

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## Attributes Should Remain Attributes

Factor models, supported by equilibrium arguments (Merton, 1973) or arbitrage arguments (Ross, 1976), are not the only key cornerstones of asset pricing theory (APT). In investment practice, multi-factor models have also become standard tools for the analysis of the risk and performance of equity portfolios. On the performance side, they allow investors and asset managers to disentangle abnormal return (or alpha) from the return explained by exposure to common rewarded risk factors. On the risk side, factor models allow us to distinguish between specific risk and systematic risk, and this decomposition can be applied to both absolute risk (volatility) and relative risk (tracking error with respect to a given benchmark).

In addition to analysing the impact of common *factors*, equity portfolio managers are also interested in analysing the role of stock-specific *attributes* in explaining differences in risk and performance across assets and portfolios. For example, it has been documented that small stocks tend to outperform large stocks (Banz, 1981) and that value stocks earn higher average returns than growth stocks (Fama and French, 1992). Moreover, stocks that have best performed over the past three to twelve months tend to outperform the past losers over the next three to twelve months (Jegadeesh and Titman, 1993).

A common explanation for these effects, which cannot be explained by Sharpe's (1964) single-factor capital asset pricing model or *CAPM* (Fama and French, 1993, 2006), is that the size and the value premia are rewards for exposure to systematic sources of risk that are not captured by the market factor. This is the motivation for the introduction of the size and value factors by

Fama and French (1993) as proxies for some unobservable underlying economic factors, perhaps related to a distressed factor. In this process, market capitalisation and the book-to-market ratio are used as criteria to sort stocks and to form long-short portfolios with positive long-term performance. In other words, what is intrinsically an attribute is turned into a factor. A similar approach is also used by Carhart (1997), who introduces a "winners minus losers" factor, also known as the momentum factor. More recently, investment and profitability factors have been introduced, so as to capture the investment and profitability effects: again Fama and French (2015) turn attributes into factors by sorting stocks on operating profit or the growth on total assets, while Hou, Xue and Zhang (2015) replace the former measure by the return on equity when constructing their profitability factor.

Overall, the standard practice of treating attributes as factors severely, and somewhat artificially, increases the number of factors to consider, especially in the case of discrete attributes. This raises a serious challenge with respect to how to perform a consistent risk and performance analysis for equity portfolios across multiple dimensions that incorporate both macro factors and micro attributes.

In this paper, we explore a novel approach to address this challenge. As opposed to artificially adding new factors to account for differences in expected returns for stocks with different attributes, we seek to maintain a parsimonious factor model and treat attributes as auxiliary variables to estimate the betas with respect to true underlying risk factors. In other words, our goal is to decompose market exposure (beta) and risk-adjusted performance (alpha) in a forward-looking way as a function of the

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firm's characteristics, so that the attributes can remain attributes in the context of a parsimonious factor model, as opposed to being artificially treated as additional factors.

Our approach is somewhat related to the literature on conditional asset pricing models (Jagannathan and Wang, 1996), who also allow the factor exposure to be a function of some state variables. One key difference is that standard conditional versions of the CAPM (e.g. Ferson and Schadt, 1996), stipulate that betas (and possibly alphas and risk premia) are functions of underlying macroeconomic factors such as the T-Bill rate, dividend yield, slope of the term structure, credit spread, etc. In contrast, we take betas to be functions of the time-varying micro attributes or characteristics of the underlying firms that are typically used to define additional factors, including in particular market capitalisation, the book-to-market ratio and past one-year performance. Of course, one could in principle regard the factor exposures as a function of stock-specific attributes and pervasive state variables.

In what follows, we introduce a formal framework for estimating these so-called *fundamental betas*, as opposed to *historical* betas, and we provide evidence of the usefulness of these fundamental betas for (i) parsimoniously embedding the sector dimension in multi-factor portfolio risk and performance analysis, (ii) building equity portfolios with controlled target factor exposure, and also (iii) explaining the cross-section of expected returns.

## Fundamental Betas as Functions of Attributes

The traditional approach to measuring the market exposure of a stock or a portfolio is to run a time-series regression of the stock (excess) returns on a market factor over a rolling window. If the joint distribution of stock and market returns were constant over time, the sample beta at date  $t - 1$  would be a consistent estimator of the conditional beta on this date, and the variation in rolling-window estimates would be due to sampling errors only. Factor exposures, however, are not constant over time and the key challenge is therefore to estimate the beta for each stock conditional on the information available to date:

$$\beta_{i,t-1} = \frac{\text{Cov}[R_{i,t}, R_{m,t} | \Phi_{t-1}]}{\text{Var}[R_{m,t} | \Phi_{t-1}]}$$

where  $R_{i,t}$  denotes the return on stock  $i$  in period  $[t - 1, t]$  in excess of risk-free rate,  $R_{m,t}$  is the excess return on the market portfolio and  $\Phi_{t-1}$  is the information set available at date  $t - 1$ . The traditional measure of conditional market exposure is the beta estimated over a sample period, but if the distributions of stock and market returns change over time, the sample estimates are not good estimators of the true conditional moments. By shifting the sample period (rolling-window estimation), one does generate time dependency in the beta, but the "historical beta" changes relatively slowly due to the overlap between estimation windows.

We introduce an alternative estimator for the conditional beta, which we name "fundamental beta" because it is defined as a function of the stock's characteristics. More specifically, we first consider the following one-factor model for stock returns, in which the alpha and the beta are functions of the three observable attributes that define

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the Fama-French-Carhart factors: market capitalisation ( $Cap_{i,t}$ ), the book-to-market ratio ( $Bmk_{i,t}$ ) and past 1-year return ( $Ret_{i,t}$ ) for the stock  $i$  at date  $t$ . Hence we have the following relations:

$$R_{i,t} = \alpha_{i,t} + \beta_{i,t} \times R_{m,t} + \varepsilon_{i,t}$$

$$\alpha_{i,t} = \theta_{\alpha,0,i} + \theta_{\alpha,Cap,i} \times Cap_{i,t} + \theta_{\alpha,Bmk,i} \times Bmk_{i,t} + \theta_{\alpha,Ret,i} \times Ret_{i,t}$$

$$\beta_{i,t} = \theta_{\beta,0,i} + \theta_{\beta,Cap,i} \times Cap_{i,t} + \theta_{\beta,Bmk,i} \times Bmk_{i,t} + \theta_{\beta,Ret,i} \times Ret_{i,t}$$

For  $N$  stocks, the model involves  $8N$  parameters  $\theta$  which tie the alphas and betas to the underlying stock characteristics. These parameters are estimated by minimising the sum of squared residuals  $\varepsilon_{i,t}^2$  over all dates and stocks in a procedure known as *pooled regression*.

Because the coefficients are independent from one stock to the other, the pooled regression is actually equivalent to  $N$  time-series regressions:

$$\text{minimise } \sum_{i,t} \varepsilon_{i,t}^2 \text{ is equivalent}$$

$$\text{to minimise } \sum_t \varepsilon_{i,t}^2 \text{ for each } i$$

Hence, the coefficients can be estimated separately for each stock, by running a time-series regression. More specifically, we regress each stock's excess return on the market return and the market return crossed with the stock's attributes. For a stock  $i$ , the regression equation takes the form:

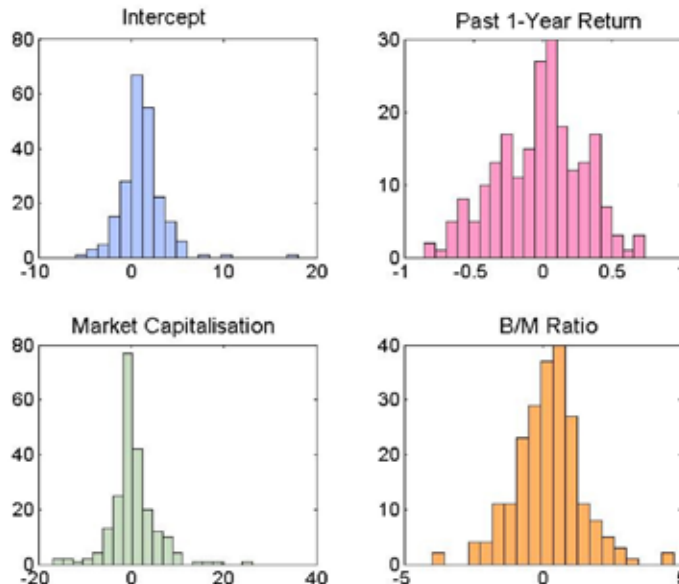
$$R_{i,t} = \theta_{\alpha,0,i} + \theta_{\alpha,Cap,i} \times (Cap_{i,t}) + \theta_{\alpha,Bmk,i} \times (Bmk_{i,t}) + \theta_{\alpha,Ret,i} \times (Ret_{i,t}) + \theta_{\beta,0,i} \times$$

$$R_{m,t} + [\theta_{\beta,Cap,i} \times (Cap_{i,t}) + \theta_{\beta,Bmk,i} \times (Bmk_{i,t}) + \theta_{\beta,Ret,i} \times (Ret_{i,t})] \times R_{m,t} + \varepsilon_{i,t}$$

Prior to the regression, each attribute is transformed into a zero mean and unit standard deviation z-score so as to avoid scale effects. The model is estimated over the S&P 500 universe with  $N = 500$  stocks and the period 2002-2015 (which corresponds to 51 quarterly returns). Hence we obtain 500 coefficients of each type (intercept, capitalisation sensitivity, book-to-market sensitivity and past return sensitivity) for the alpha, and 500 others for the beta. Exhibit 1 displays the distributions of the four coefficients that appear in the

*Exhibit 1: Distributions of Coefficients in the More Flexible One-Factor Model*

*The coefficients are estimated through time-series regressions for each of the 500 stocks from the S&P 500 universe with quarterly stock returns, z-score attributes and market returns from Ken French's library over the period 2002-2015. Attributes come from the ERI Scientific Beta US database and are updated quarterly.*





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decomposition of the fundamental beta, and suggests that there is a substantial dispersion in the estimates across the 500 stocks.

## Sector in Multi-Dimensional Portfolio Analysis with Fundamental Betas

Risk and performance analysis for equity portfolios is most often performed according to one single dimension, typically based on sector, country or factor decompositions. In reality, risk and performance of a portfolio can be explained by a combination of several such dimensions, and the question arises to assess, for example, what the marginal contributions of various sectors are in addition to stock-specific attributes to the performance and risk of a given equity portfolio. The fundamental beta approach can be used for this purpose, provided that one introduces a sector effect in the specification of the conditional alpha and beta. This is done by replacing the stock-specific constants  $\theta_{\alpha,0,i}$  and  $\theta_{\beta,0,i}$  by sector-specific terms, which only depend

on the sector of stock  $i$ . The method can be easily extended to handle country effects in addition to sector effects. Formally, the model reads:

$$R_{i,t} = \alpha_{i,t} + \beta_{i,t} \times R_{m,t} + \varepsilon_{i,t}$$

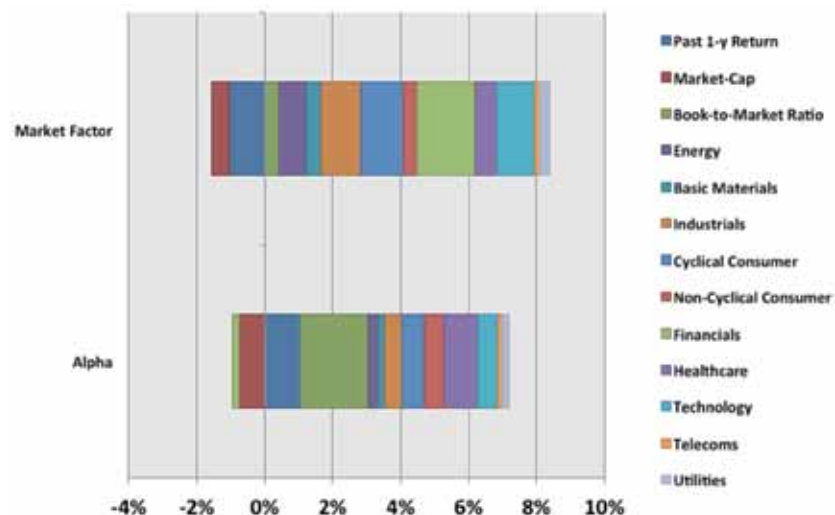
$$\alpha_{i,t} = \theta_{\alpha,0,s(i)} + \theta_{\alpha,Cap,i} \times Cap_{i,t} + \theta_{\alpha,Bmk,i} \times Bmk_{i,t} + \theta_{\alpha,Ret,i} \times Ret_{i,t}$$

$$\beta_{i,t} = \theta_{\beta,0,s(i)} + \theta_{\beta,Cap,i} \times Cap_{i,t} + \theta_{\beta,Bmk,i} \times Bmk_{i,t} + \theta_{\beta,Ret,i} \times Ret_{i,t}$$

This model can be used to decompose the expected return and the variance of a portfolio conditional on the current weights and constituents' characteristics. In Exhibit 2, we show an application of this method to the analysis of the expected performance a broad equally-weighted portfolio of US stocks. At the first level, expected return is broken into a systematic part – which comes from the market exposure – and an abnormal part. Each of these two components is further decomposed into contributions from sectors and continuous attributes.

*Exhibit 2: Absolute Performance Decomposition of the EW S&P 500 Index on Market Factor with Fundamental Alpha and Fundamental Beta*

*The coefficients of the one-factor model are estimated with a pooled regression of the 500 stocks from the S&P 500 universe. Data is quarterly and spans the period 2002-2015, and market returns are from Ken French's library. Attributes (capitalisation, book-to-market and past one-year return) and sector classification come from the ERI Scientific Beta US database and are updated quarterly. We use formula 3.2 to perform the performance attribution.*



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As we can see, the book-to-market ratio has a positive impact both on market exposure and alpha, suggesting that a higher book-to-market ratio implies higher abnormal performance and market exposure, while the past one-year return has a positive impact on alpha but a negative impact on market exposure. Finally market capitalisation has a negative impact on both alpha and market exposure, confirming that large caps tend to have smaller abnormal performance and market exposure. Within market factor exposure and alpha contributions, some sectors have larger contributions, such as Financials, Industrials and Cyclical Consumer for market exposure and Healthcare for abnormal performance.

### Targeting Market Neutrality with Fundamental Betas

We then compare the fundamental and the rolling-window betas as estimators of the conditional beta by constructing market-neutral portfolios based on the two methods. We show that the fundamental method results in more accurate estimates of market exposures, since the portfolios constructed in this way achieve better ex-post market neutrality compared to those in which the beta was estimated by running rolling-window regressions, which tend to smooth variations over time thereby slowing down the diffusion of new information

in the beta. In contrast, the fundamental beta is an explicit function of the most recent values of the stock's characteristics, and as such is more forward-looking in nature.

In order to achieve more robustness in the results, we do not conduct the comparison for a single universe, but we repeat it for 1,000 random universes of 30 stocks picked among the 218 that remained in the S&P 500 universe between 2002 and 2015. Hence we have 1,000 random baskets of 30 stocks, and, for each basket, we compute the two market-neutral portfolios.

Exhibit 3 shows that portfolios based on fundamental beta achieve, on average, better market neutrality (corresponding to a target beta equal to 1) than those based on time-varying historical beta, with an in-sample beta of 0.925 versus 0.869 on average across the 1,000 universes. We observe the same phenomenon in terms of correlation with an average market correlation of 0.914 for portfolios based on fundamental betas, versus 0.862 for the portfolios based on historical time-varying beta.

At each date, we also compute the 1,000 absolute differences between the 5-year rolling-window beta and the target of, and the results are reported in Exhibit 4. The historical method exhibits the largest

*Exhibit 3: Targeting Beta Neutrality for Maximum Deconcentration Portfolios Based on Fundamental and Time-Varying Historical Betas (2002-2015)*

*1,000 maximum deconcentration portfolios of 30 random stocks subject to a beta neutrality constraint are constructed by using the rolling-window or the fundamental betas. The 30 stocks are picked among the 218 that remained in the S&P 500 universe for the period 2002-2015, and the portfolios are rebalanced every quarter. The control regression on Ken French's market factor is done using quarterly returns over the period 2002-2015. Market betas and correlations with the market return are computed for each portfolio over the period 2002-2015 and are averaged across the 1,000 universes. Also reported are the standard deviations of the beta and the correlation over the 1,000 universes.*

	Out-of-Sample Market beta		Out-of-Sample Market correlation	
	Mean	Standard Deviation	Mean	Standard Deviation
Historical	0.869	0.032	0.862	0.025
Fundamental	0.925	0.035	0.914	0.020

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deviation levels with respect to the target, with a number of dates (such as March 1996, December 2005 or March 2007) where the relative error exceeds 60%! In comparison, the fundamental method leads to much lower extreme differences between target and realised factor exposures, thus suggesting that this methodology allows for the error in the estimation of the conditional betas to be reduced versus what can be achieved with the classical rolling-window approach.

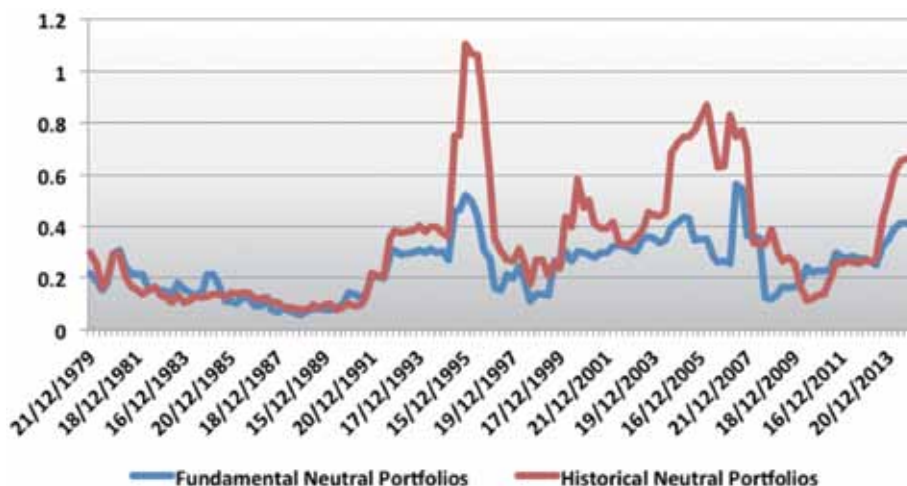
### Fundamental Betas and the Cross-Section of Expected Returns

The main goal of an asset pricing model is to explain the differences in expected returns across assets through the differences in their exposures to a set of pricing factors. It is well known that the standard CAPM largely misses this goal, given its inability to explain effects such as size, value and momentum. In this subsection, we investigate whether the fundamental CAPM introduced in Section 2.2.3 is more successful from this perspective. To this end,

we conduct formal asset pricing tests by using Fama and MacBeth method (1973). There are two statistics of interest in the output of these tests. The first one is the average alpha of the test portfolios, which measures the fraction of the expected return that is not explained by the model. The second set of indicators is the set of factor premia estimates, which should have plausible values.

More specifically, we test two versions of the conditional CAPM based on fundamental betas, one with a constant market premium and one with a time-varying market premium. The latter approach is more realistic since it is well documented that some variables, including notably the dividend yield and the default spread, have predictive power over stock returns, at least over long horizons – see Fama and French (1988, 1989), Hodrick (1992), Menzly, Santos and Veronesi (2004). Introducing a time-varying market premium implies that the unconditional expected return of a stock depends not only on its average conditional beta but also on the covariance between the

*Exhibit 4: Largest Distance to 1 for the Portfolio Realised Beta across 1,000 Universes*  
1,000 maximum deconcentration portfolios of 30 random stocks subject to a beta neutrality constraint are constructed by using the rolling-window or the fundamental betas. The 30 stocks are picked among the 71 that remained in the S&P 500 universe for the period 1970-2015, and the portfolios are rebalanced every quarter. The control regression on Ken French's market factor is done on a 5-year rolling window of quarterly returns. For each window, the Exhibit shows the largest distance to 1 computed over the 1,000 universes.



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conditional beta and the conditional market premium (Jagannathan and Wang, 1996).

In Exhibit 5, we compare the distributions of alphas across the 30 portfolios for the four competing models. These results suggest that the parsimonious fundamental conditional CAPM with constant market premium is substantially more effective than the standard static CAPM for explaining differences in expected returns, with an average alpha that is dramatically reduced from 5.04% down to 1.69%. Remarkably, this model performs as well as the less parsimonious Fama-French-Carhart 4 factor model.

The results reported in the exhibit also suggest that accounting for the covariance term between the conditional beta and the conditional market premium further improves the ability of the fundamental CAPM to explain the returns of portfolios sorted on size, book-to-market or short-term past returns with respect to the case where the premium is constant. Furthermore, the average alpha obtained with this model is almost half the value obtained with Fama-French-Carhart model, suggesting that a conditional CAPM based on fundamental betas and a time-varying risk premium can capture the size, value and momentum effects better than the Fama-French-Carhart model, and this

without the help of additional ad-hoc factors.

### Parsimonious and Forward-Looking Risk Indicators for Equity Portfolios

Multi-factor models are standard tools for analysing the performance and the risk of equity portfolios. In the standard Fama-French-Carhart model, size, value and momentum factors are constructed by first sorting stocks on an attribute (market capitalisation, the book-to-market ratio or past short-term return), then by taking the excess return of the long leg over the short leg. While these models are substantially more successful than the standard CAPM at explaining cross-sectional differences in expected returns, the empirical link between certain characteristics and average returns can always be accounted for by introducing new ad-hoc factors in an asset pricing model. In the end, numerous patterns have been identified in stock returns, thus raising concerns about a potential inflation in the number of long-short factors and their overlap.

Our analysis suggests another meaningful approach for explaining the cross-section of expected returns, which consists in treating attributes of stocks as instrumental variables to estimate the exposure with respect to a parsimonious set of factors.

*Exhibit 5: Alphas Distribution over the Cross-Section of Sorted Portfolios*

*This exhibit provides the distribution of the estimated alphas for 30 portfolios sorted on size, book-to-market or past one-year return. These alphas are obtained by performing Fama-MacBeth regressions for three pricing models. The fourth row shows the distribution of alphas obtained in the conditional CAPM with fundamental beta and a time-varying market premium. The fundamental beta is a function of the constituents' attributes. Regressions are done on the period 1973-2015. The last column shows the average t-statistics across alphas.*

	Mean	Std	1st Quartile	Median	Third Quartile	Mean Corrected T-stats
Static CAPM	5.04%	2.74%	3.35%	4.95%	6.35%	1.66
Carhart Model	2.87%	1.06%	2.35%	2.66%	3.57%	0.89
Fundamental CAPM	2.86%	2.72%	1.08%	2.76%	4.19%	0.79
Fundamental CAPM with Time-Varying Market Factor	1.69%	2.70%	-0.27%	1.54%	3.39%	0.71

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As an illustration, we have focused on the conditional CAPM one-factor model, and we estimate a time-varying beta that is explicitly given by a linear function of the very same characteristics that define the three Fama-French-Carhart factors. We show that a conditional CAPM based on this *fundamental* beta can capture the size, value and momentum effects as well as the Fama-French-Carhart model, but without the help of additional factors. The pricing errors are further reduced by introducing a time-varying market premium, which introduces the cyclical covariation between fundamental betas and the market risk premium as a driver of expected returns. The fundamental beta also provides an alternative measure for the true unknown value of the conditional beta. This estimate is a function of observable variables and is not subject to the artificial smoothing effect that impacts betas estimated by a rolling-window regression analysis. Since the fundamental beta immediately responds to changes in the value of a stock's attributes, they can be used to more effectively assess the impact of a change in the portfolio composition on the factor exposure. We illustrate these benefits by constructing market-neutral portfolios based on the fundamental and the rolling-window methods, and we show that the former approach achieves better out-of-sample neutrality. Interestingly, this approach can be extended in a straightforward manner from a single-factor model to a multi-factor model, thus allowing exposure to a variety of underlying systematic macro factors to depend upon the micro characteristics of the firm.

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# Introduction



# Introduction

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Factor models seek to explain the differences in expected returns across assets by their exposure to a set of common factors, which represents the risk factors that are of concern to investors given that they require compensation in the form of higher expected returns for bearing exposure to these factors. Historically, the Capital Asset Pricing Model (CAPM) of Treynor (1961) and Sharpe (1963, 1964) was the first of these factor models. It explains differences in expected returns across securities by their respective sensitivities to a single factor, which is the return on the market portfolio. In 1976, Steve Ross introduced the Arbitrage Pricing Theory for the purpose of valuing assets under the assumptions that there was no arbitrage and that asset returns could be decomposed into a systematic part and an idiosyncratic part. An independent theory of multi-factor asset pricing models has been developed by Merton (1973), with the Intertemporal CAPM. In the ICAPM, expected returns are determined by the exposures to those factors that drive conditional expected returns and volatilities.

1 - In fact, this statement appears to depend on the period under study. For instance, Fama and French (2006) show that the CAPM fails to explain the value premium between 1963 and 2004, since value stocks have lower betas than growth stocks. However, in the period from 1926 to 1963, the CAPM accounts for the value premium. Rejection of the CAPM over the whole period thus seems to be due to the second half of the sample.

According to most empirical studies, the CAPM in its original form has very limited success in capturing differences in expected returns. The positive relationship between the expected return and the market beta is seriously challenged by the existence of a low beta anomaly (Frazzini and Pedersen, 2014), and it has long been documented that market exposure is not the only determinant of expected returns. For instance, small stocks tend to outperform large stocks (Banz, 1981; Van Dijk, 2011) and value stocks – stocks with a high book-to-market ratio – earn higher average returns than growth stocks (Statman, 1980; Fama and

French, 1992). Moreover, stocks that have best performed over the past three to twelve months tend to outperform the past losers over the next three to twelve months (Jegadeesh and Titman, 1993). None of these effects can be explained by the traditional CAPM, as the return spreads cannot be justified by differences in market exposures (Fama and French, 1993, 2006).<sup>1</sup>

A common explanation for these effects is that the size and the value premia are rewards for exposure to systematic sources of risk that are not captured by the market factor. This is the motivation for the introduction of the size and value factors by Fama and French (1993). The size factor is defined as the excess return of a portfolio of small stocks over a portfolio of large stocks, and the value factor is defined as the excess return of value over growth stocks. In this process, market capitalisation and the book-to-market ratio are used as criteria to sort stocks and to form long-short portfolios with positive long-term performance. This approach has been extended to the momentum factor by Carhart (1997), who shows that the continuation of past short-term performance is not accounted for by the Fama-French three-factor model (Fama and French, 1996), but it can be somewhat tautologically explained by introducing a “winners minus losers” portfolio as a fourth factor in an augmented version of the Fama-French model. More recently, the investment and profitability factors have been introduced, so as to capture the investment and profitability effects: Fama and French (2015) sort stocks on operating profit or the growth on total assets, and Hou, Xue and Zhang (2015) replace the former measure by the return on equity. There is some overlap between



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all these factors, as suggested by Hou, Xue and Zhang (2015), who show that the book-to-market effect is predicted by a four-factor model with the market, the size factor and the investment and profitability factors.

Multi-factor models have thus become standard tools for the analysis of the risk and performance of equity portfolios. On the performance side, they allow us to disentangle abnormal returns (alpha) from the returns explained by exposure to common risk factors. The alpha component is interpreted as "abnormal return" because it should be (statistically not different from) zero if factor exposures were able to explain any difference between expected returns. Thus, a non-zero alpha reveals either misspecification of the factor model, from which relevant factors have been omitted, or genuine skill of the manager who was able to exploit pricing anomalies. On the risk side, factor models allow us to distinguish between specific risk and systematic risk, and this decomposition can be applied to both absolute risk (volatility) and relative risk (tracking error with respect to a benchmark).<sup>2</sup>

The performance and risk decomposition of a portfolio across factors is receiving increasing attention from sophisticated investors. Recent research (Ang, Goetzmann and Schaefer, 2009; Ang, 2014) has highlighted that risk and allocation decisions could be best expressed in terms of rewarded risk factors, as opposed to standard asset class decompositions. Bhansali et al. (2012) evaluate the benefits of using a factor-based diversification measure over asset-based measures. They use a principal component analysis to extract two risk factors driven by global

growth and global inflation. They show that asset-based risk parity portfolios can often concentrate too much in just one component of risk exposures, particularly equity risk, in contrast to factor-based risk parity which allows a more robust risk diversification. A related argument is made by Carli, Deguest and Martellini (2014), who emphasise the importance of reasoning in terms of uncorrelated factors to assess the degree of diversification of a portfolio. Finally, a recent strand of research has started to look at factor investing as a tool for portfolio construction or asset allocation. In this approach, it is the factors that are regarded as the constituents of a portfolio – see Martellini and Milhau (2015) and Maeso and Martellini (2016).

Performance and risk attribution models used by practitioners, such as the Barra models, often include "factors" other than those borrowed from asset pricing theory. Typical examples are sector and country factors. The question therefore arises to assess what exactly are the marginal contributions of the various dimensions to the return and the volatility of a given equity portfolio. A straightforward procedure is to introduce the new factors as additional regressors in the econometric model. Menchero and Poduri (2008) develop a multi-factor model in which the set of pricing factors is extended with "custom factors". We apply this method to the multi-dimensional analysis of various equity portfolios, and we propose an alternative, more parsimonious, holding-based method (as opposed to a purely return-based) approach when information about portfolio holdings is available. Overall, it must be acknowledged that increasing the number of factors raises concerns about their potential overlap: after all,

<sup>2</sup> - In Section 1 of this paper, we apply these decomposition methods to the analysis of ex-post performance, volatility and tracking error of US equity mutual funds. We also review multi-factor models commonly used by practitioners, such as Barra models, which include sector and country attributes in addition to risk factors.

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sorts based on sector, country, size or book-to-market are only different ways of segmenting the same universe. Hence, it is desirable to have a decomposition method that keeps the number of factors reasonably low while being flexible enough to handle a wide variety of attributes.

Furthermore, the risk-based explanation of the size, value and momentum effects, and the need for the related factors, is debated. First, there is no consensual interpretation of the size, value and momentum factors as risk factors in the sense of asset pricing theory. Indeed, the factors that can explain differences in expected returns are those that affect the marginal expected utility from consumption in consumption-based asset pricing models, and those that determine the comovements between stocks in models based on the APT. However, there is no unique and definitive explanation of why small, value and winner stocks would be more exposed to such systematic risk than large, growth and loser stocks. Second, all these effects can be explained without the help of additional factors. For instance, Daniel and Titman (1997) argue that expected returns depend on the size and the book-to-market ratio rather than the exposure to the Fama-French long-short factors. They also reject the interpretation of these factors as "common risk factors", arguing that the high correlations within small or value stocks simply reveal similarities in the firms' activities. It has also been documented that behavioural models, in which investors display excessive optimism or reluctance with respect to some stocks, can explain the observed outperformance of small, value or winner stocks (Merton, 1987; Lakonishok, Shleifer and Vishny, 1994; and Hong and Stein, 1999).

A more recent literature has re-assessed the ability of the CAPM to explain the anomalies, by focusing on a conditional version of the model. In fact, traditional measures of alpha and beta in the CAPM are conducted as if these quantities were constant over time, by performing time-series regressions of stock returns on a market factor. As a result, it is an unconditional version of the CAPM that is tested. The conditional version of the model posits that conditional expected returns are linearly related to conditional market betas, the slope being the conditional market premium. Different specifications have been studied in the literature. Gibbons and Ferson (1985) allow for changing expected returns but assume constant betas, while Harvey (1989) emphasises the need for time-varying conditional covariances between stocks and the market factor. Jagannathan and Wang (1996) introduce both time-varying betas and a time-varying market premium. A crucial point in empirical studies is how the set of conditioning information is specified. Ferson and Schadt (1996) let the conditional betas be a function of lagged macroeconomic variables, namely the T-Bill rate, the dividend yield, the slope of the term structure, the spread of the corporate bond market, plus a dummy variable for the January effect. Jagannathan and Wang (1996) model the conditional market premium as a function of the default spread in the bond market, but do not explicitly model the conditional betas. Lettau and Ludvigson (2001) use the log consumption-wealth ratio. Lewellen and Nagel (2006) do not specify a set of conditioning variables, and they estimate alphas and betas over rolling windows, assuming that conditional alphas and betas are stable over the estimation window (one month

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or one quarter). Ang and Chen (2007) have also no explicit model for the betas, but they treat them as latent variables to be estimated by filtering techniques. The empirical success of these models is mixed. Harvey (1989) finds that the data rejects the model's restrictions, and Lewellen and Nagel (2006) conclude that the conditional CAPM performs no better than the unconditional one in explaining the book-to-market and the momentum effects. On the other hand, Ang and Chen (2007) report that the alpha of the long-short value-growth strategy is almost insignificant in a conditional model.

The measurement of conditional betas is not only essential for researchers testing asset pricing models, but also for portfolio managers who want to implement an allocation consistent with their views on factor returns. For instance, a fund manager who anticipates a bear market will seek to decrease the beta of his or her portfolio, and one who expects temporary reversal of the momentum effect will underweight past winners. For practical applications, it is clearly useful to have an expression of the conditional beta as a function of observable variables, as opposed to having it extracted by filtering methods. The main contribution of this paper is to propose an alternative specification for the conditional market beta, as a function of the very same characteristics that define the Fama-French and Carhart factors, and we call it a "fundamental beta". We have the same linear specification for the beta as Ferson and Schadt (1996), but we replace the macroeconomic variables by the market capitalisation, the book-to-market ratio and the past one-year return. This choice of variables is motivated by several reasons. First, it makes intuitive sense that the microeconomic characteristics

of a firm matter to explain its exposure to market risk. For instance, Beaver, Kettler and Scholes (1970) find a high degree of contemporaneous association between the market beta and accounting measures such as the dividends-to-earnings, growth, leverage, liquidity, asset size, variability of earnings and covariability of earnings within the context of 'intrinsic value' analysis. By representing the beta as an explicit function of these characteristics, one is able to immediately incorporate the most recent information about a stock. If the time-varying beta was estimated through a rolling-window regression, this information would be reflected with a lag, since this type of regression by construction tends to smooth variations over time. Second, we choose characteristics that correspond to well-documented and economically grounded patterns in equity returns, namely the size, the value and the momentum effects. In particular, we are interested in testing whether variations in the fundamental beta across stocks are consistent with an expected outperformance of small, value and winner stocks. We estimate three specifications for the fundamental beta, by employing pooled regression techniques, as suggested by Hoechle, Schmidt and Zimmermann (2015). In recent work, these authors introduce a Generalised Calendar Time method, which allows them to represent alphas and betas of individual stocks as a function of discrete or continuous characteristics. With this powerful technique, it is thus possible to let the alpha and beta of each stock depend on the sector as well as the market capitalisation, the book-to-market ratio and the past recent return.

We then present three applications of the fundamental beta. In the first one we use

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the fundamental beta approach to include sector effects along with observable attributes that define the Fama-French and Carhart factors in the analysis of expected return and volatility of a portfolio. This provides a parsimonious alternative to the decomposition methods that introduce dedicated factors for additional attributes such as sector and country classifications. The second application of the fundamental beta approach is the construction of portfolios with a target factor exposure. We compare the out-of-sample beta of a portfolio constructed by the fundamental method with that of a portfolio constructed through the rolling-window approach. For both portfolios, the out-of-sample beta is estimated by performing a full-period regression on the market, in order to have a consistent comparison criterion. This protocol is similar to that employed to compare competing estimators of the covariance matrix, when minimum variance portfolios are constructed with various estimators and their out-of-sample variances are computed. We find that out-of-sample, the portfolio constructed on the basis of fundamental betas is indeed closer to neutrality (defined as a target value of 1) compared to the portfolio constructed on the basis of the rolling-window betas. The last application that we consider is a "fundamental CAPM", which is a form of conditional CAPM where the conditional beta of a stock depends on its characteristics. We compute the alphas of portfolios sorted on size, book-to-market and past one-year return by performing cross-sectional regressions of the Fama and MacBeth (1973) type. We confirm that the unconditional CAPM yields large pricing errors for these portfolios, and we find that the conditional version of the model has roughly the same pricing

errors as the less parsimonious four-factor model of Carhart (1997). We also show that the magnitude of the alphas is further reduced by introducing a time-varying market premium, as in Jagannathan and Wang (1996).

The rest of the paper is organised as follows. Section 1 contains a reminder on factor models in asset pricing theory and on the empirical models developed by Fama and French (1993) by Carhart (1997). We also discuss the question of performance and risk attribution, first with respect to the Fama-French and Carhart factors, and then to the same set of factors extended with sectors. We also illustrate these methods on various portfolios of US stocks. In Section 2, we define the fundamental betas and we discuss the estimation procedure in detail. Section 3 presents three applications of the fundamental beta approach to embed the sector dimension in a multi-dimensional performance and risk analysis, the construction of the market-neutral portfolios and the pricing of portfolios sorted on size, book-to-market and past short-term return, respectively. Section 4 concludes.

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In this section, we start with a reminder on the standard factor models that have been developed in the academic literature. The purpose of these models is to identify the systematic risk factors that explain the differences in expected returns across financial assets. Historically, this has been achieved through theoretical analysis based on economic or statistical arguments, or through empirical studies of the determinants of expected returns. We then show how one of these models, namely the four-factor model of Carhart (1997), can be used to decompose the ex-post performance and volatility of a portfolio. However, industry practices often rely on models that involve a much larger number of factors, such as Barra-type models, to analyse performance and risk. We review these models below, by giving attention to the multi-collinearity issues raised by the simultaneous inclusion of numerous factors, and we provide examples of multi-dimensional decomposition across risk factors and sectors on US equity portfolios.

3 - Empirical evidence has linked variations in the cross-section of stock returns to firm characteristics such as market capitalisation and book-to-market values (Fama and French, 1992, 1993) and short term continuation effect in stock returns (Carhart, 1997).

## 1.1 Factor Model Theory

This section is a reminder on the standard factor models that have been developed in the academic literature. A complete presentation of the theory can be found in Cochrane (2005), who relates the factors to the stochastic discount factor. While our paper is focused on equity portfolios, factor models can be in principle developed for other asset classes, such as bonds and commodities.

### 1.1.1 The Single-Factor Model

The CAPM of Sharpe (1963, 1964) and Treynor (1961) is a model for pricing an individual security or portfolio. In this model, the differences in expected returns across securities are explained by their respective sensitivities to a single factor,

which is the return on the market portfolio. At equilibrium, the returns on assets less the risk-free rate are proportional to their market beta. Mathematically, the CAPM relationship at equilibrium is written as follows:

$$E(R_i) = B_i \times E(R_m) \quad (1.1)$$

where  $R_i$  denotes the excess return on asset  $i$  (in excess of the risk-free rate),  $R_m$  denotes the excess return on the market portfolio.  $B_i$ , the exposure to the market factor, is defined by  $B_i = \frac{\sigma_{i,m}}{\sigma_m^2}$  where  $\sigma_{i,m}$  denotes the covariance between asset  $i$  and the market portfolio and  $\sigma_m^2$  denotes the variance of the market portfolio. The correct measure of risk for an individual asset is therefore the beta, and the reward per unit of risk taken is called the risk premium. The asset betas can be aggregated: the beta of a portfolio is obtained as a linear combination of the betas of the assets that make up the portfolio. In this model, the asset beta is the only driver of the expected return on a stock. Accurate measurement of market exposure is critical for important issues such as performance and risk measurement.

However, the model is based on very strong theoretical assumptions which are not satisfied by the market in practice. The model assumes, inter alia, that investors have the same horizon and expectations. Moreover, the prediction of an increasing link between the expected return and the market beta is not validated by examination of the data: the relation tends to be actually decreasing (Frazzini and Pedersen, 2014), and there are a number of empirical regularities in expected returns, such as the size, value and momentum effects, that cannot be explained with the CAPM and thus constitute anomalies for the model.<sup>3</sup>

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## 1.1.2 Multi-Factor Models and the Arbitrage Pricing Theory

The current consensus tends towards the idea that a single factor is not sufficient for explaining the cross-section of expected returns. In 1976, Ross introduced the Arbitrage Pricing Theory for the purpose of valuing assets under the assumptions that there is no arbitrage and that asset returns can be decomposed into a systematic part and an idiosyncratic part. Unlike in the CAPM, no assumption is made regarding the investment decisions made by individual agents. This model is also linear and employs  $K$  factors and it nests the CAPM as a special case.

The APT model postulates that a linear relationship exists between the realised returns of the assets and the  $K$  common factors:

$$R_{i,t} = E(R_i) + \sum_{k=1}^K B_{i,k} F_{k,t} + \varepsilon_{i,t} \quad (1.2)$$

- $R_{i,t}$  denotes the excess return for asset  $i$  in period  $t$
- $E(R_i)$  denotes the expected return for asset  $i$
- $B_{i,k}$  denotes the sensitivity (or exposure) of asset  $i$  to factor  $k$
- $F_{k,t}$  denotes the return on factor  $k$  at period  $t$  with  $E(F_k)=0$
- $\varepsilon_{i,t}$  denotes the residual (or specific) return of asset  $i$ , i.e. the fraction of return that is not explained by the factors, with  $E(\varepsilon_i)=0$ .

The residuals returns of the different assets are assumed to be uncorrelated from each other and uncorrelated from the factors.

We therefore have:

- $\text{cov}(\varepsilon_i, \varepsilon_j) = 0$ , for  $i \neq j$
- $\text{cov}(\varepsilon_i, F_k) = 0$ , for all  $i$  and  $k$ .

Arbitrage reasoning then allows us to end up with the following equilibrium relationship:

$$E(R_i) = \sum_{k=1}^K B_{i,k} \lambda_k \quad (1.3)$$

where  $r_f$  denotes the risk-free rate. This relationship explains the average asset return as a function of the exposures to different risk factors and the market's remuneration for those factors. The  $B_{i,k}$  are the sensitivities to the factors (factor loadings) and can be obtained by regressing realised excess returns on factors.  $\lambda_k$  is interpreted as the factor  $k$  risk premium.

Multi-factor models do not explicitly indicate the number or nature of the factors. We discuss this question in Section 1.1.3.

## 1.1.3 Common Factors versus Pricing Factors

The APT suggests two ways for the search for meaningful factors. First, the requirement of having zero, or at least low, correlation between idiosyncratic risks calls for identifying the common sources of risk in returns. From a statistical standpoint, this is equivalent to having high  $R^2$  and significant betas in the regression Equation (1.2). Second, the linear relationship between expected returns and the betas implies that factor exposures should have explanatory power for the cross-section of average returns. This links the model to the literature on empirical patterns arising in stock returns, the most notorious of which being the size, value, momentum, low volatility and profitability and investment effects.

The two sets of factors do not necessarily coincide. For instance, Chan, Karceski and Lakonishok (1998) evaluate the performance of various proposed factors in capturing return comovements which provide sources of portfolio risk. They



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show that the factors that drive return comovements may not coincide with the factors that explain the behaviour of expected returns and for this reason are not priced. In the aim to decompose both performance and risk portfolio, we must not overlook common factors which are as important as priced factors. The authors argue that if the common variation in asset returns can be explained by a small set of underlying factors, then these factors serve as candidates for the sources of priced risk (pricing factors), as the Fama-French factors are. Conversely, in accordance to APT, pricing factors are considered to be common factors because they capture return comovements. But not all common factors should be considered as pricing factors, because they may be not priced.

4 - But lose more than half of this return in the following 24 months.

Two major techniques are opposed in identifying the common factors for the assets. A first method, which is called an exogenous or explicit factor method, consists of determining the factors in advance: more often than not, these factors are fundamental factors (see below). A second method, which is called an endogenous or implicit factor method, involves extracting the factors directly from the historical returns, with the help of methods drawn from factor analysis. Although the second method guarantees by construction that the factors have ability to explain common variation in returns (at least in the sample), the problem of interpreting the factors is posed.

Arguably the mostly used explicit factors today are fundamental factors. Fundamental factors capture stock characteristics such as industry membership, country membership, valuation ratios, and technical indicators, to name only a few. The most popular factors today – Value, Growth, Size, Momentum – have been studied for

decades as part of the academic asset pricing literature and the practitioner risk factor modelling research. Rosenberg and Marathe (1976) were among the first to describe the importance of these stock traits in explaining stock returns, leading to the creation of the multi-factor Barra risk models. Later, one of the best known efforts in this space came from Eugene Fama and Kenneth French in the early 1990s. Fama and French (1992, 1993) put forward a model explaining US equity market returns with three factors: the “market” (based on the traditional CAPM model), the size factor (large- vs. small-capitalisation stocks) and the value factor (low vs. high book-to-market). The “Fama-French” model has become a standard model for performance evaluation and a necessary benchmark for any multi-factor asset pricing model. Carhart four-factor model (1997) is an extension of the Fama-French three-factor model including a momentum factor to account for the short-term continuation effect in stock returns: this pattern was identified by Jegadeesh and Titman (1993), who show that stocks that perform the best over a three- to twelve-month period tend to continue to outperform the losers over the subsequent three to twelve months. Stocks that perform the best generate an average cumulative return of 9.5% over the next 12 months.<sup>4</sup>

Empirical studies show that these factors have exhibited excess returns above the market. For instance, the seminal Fama and French (1992) study found that the average small cap portfolio (averaged across all sorted book-to-market portfolios) earned monthly returns of 1.47% in contrast to the average large cap portfolio's returns of 0.90% from July 1962 to December 1990. Similarly, the average high book-to-market portfolio (across all sorted size portfolios) earned 1.63% monthly returns compared to



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0.64% for the average low book-to-market portfolios.

## 1.1.4 Carhart's Four-Factor Model

Using a four-factor model, Daniel et al. (1997) studied fund performance and concluded that performance persistence in funds is due to the use of momentum strategies by the fund managers, rather than the managers being particularly skilful at picking winning stocks. This momentum anomaly stays unexplained by the Fama and French 3-factor model. Carhart (1997) and Chan, Chen and Lakonishok (2002) report that the four-factor model shows significant improvement over the single market factor CAPM model in explaining equity portfolio performance.

This empirical model is an extension of Fama and French's three-factor model that includes a momentum factor:

$$E(R_i) = B_{i,market}E(R_m) + B_{i,size}E(SMB) + B_{i,value}E(HML) + B_{i,mom}E(WML)$$

Where

- $E(R_i)$  denotes the expected return for asset  $i$
- $B_{i,k}$  denotes the  $k$ -th factor loadings
- $E(R_m)$  denotes the expected return of the market portfolio
- $SMB$  (small minus big) denotes the difference between returns on two portfolios: a small-capitalisation portfolio and a large-capitalisation portfolio
- $HML$  (high minus low) denotes the difference between returns on two portfolios: a portfolio with a high book-to-market ratio and a portfolio with a low book-to-market ratio
- $WML$  (winner minus losers) denotes the difference between the average of the highest returns and the average of the lowest returns from the previous year.

In the remainder of the Section 1, we will use the Fama-French-Carhart model, which is a standard choice in practice. In this model the returns on a given equity portfolio can be decomposed in five components: market factor, value factor, size factor, momentum factor, and a residual component.<sup>5</sup> The betas can be estimated by performing the following regression model for  $R_{i,t}$ , the return on stock  $i$  at period  $t$  in excess of risk-free rate:

$$R_{i,t} = \alpha_i + B_{i,market}(R_{m,t}) + B_{i,size}(SMB_t) + B_{i,value}(HML_t) + B_{i,mom}(WML_t) + \varepsilon_{i,t} \quad (1.4)$$

## 1.2 Portfolio Risk and Performance Analysis with Factors

It is widely agreed that factor allocation accounts for a large part of the variability in the return on an investor's portfolio. Indeed recent research (Ang, Goetzmann and Schaefer, 2009) has highlighted that risk and allocation decisions could be best expressed in terms of rewarded risk factors, as opposed to standard asset class decompositions. Risk reporting is increasingly regarded by sophisticated investors as an important ingredient in their decision making process. On the performance side, factor models disentangle the abnormal return (alpha) and normal return (beta). On the risk side, factor models allow us to distinguish between specific risk and systematic risk, from either an absolute or relative risk perspectives. Because of the non-linearity of portfolio risk decomposition with the volatility as risk measure, we explore some methods to handle this issue and give attention to the multi-collinearity issues raised by the simultaneous inclusion of several factors. We illustrate these decompositions with four US equity mutual funds by performing their risk and performance analysis with the Carhart four-factor model.

5 - When assessing the relative importance of the market factor versus long-short factors, one typically obtains for well-diversified portfolios a strong domination of the market factor. In the limit, if the equity portfolio to be analysed is the market index used as a proxy for the market factor, then the contribution of the market factor will be trivially 100%, while the contribution of other factors will be measured as zero. In case one would like to make statements about factor exposures for a cap-weighted market index, it is possible to take equally-weighted factors (for the market factor as well as other factors) to avoid such trivial statements.

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## 1.2.1 Expected Return Decomposition

Factor allocation is generally defined as the exposures of an investor's portfolio to a number of risk factors. These risk factors are rewarded over long period (for pricing factors) and the estimation of portfolio exposures to these factors augmented with the risk-adjusted performance enable us to decompose the portfolio expected return over the period. In a factor-based decomposition, a multi-factor model allows to measure the impact on performance of the portfolio manager's decision to implement biases towards certain factors such as value, size and momentum.

Using expression (1.3), we can decompose stock expected return through its factor exposures. We obtain, with the Carhart model, the following performance decomposition for the stock or portfolio  $i$ :

$$E(R_i) = \alpha_i + B_{i,market} \lambda_{market} + B_{i,size} \lambda_{size} + B_{i,value} \lambda_{value} + B_{i,mom} \lambda_{mom}$$

The risk premia are given by:

$$\lambda_k = E(F_k) \text{ for long-short factors or } \lambda_k = E(F_k) - r_f \text{ for long-only factors.}$$

The decomposition can also be applied to ex-post average return  $\bar{R}_i$  which is an estimator for the expected return. In this case, we first estimate the four factor loadings and the alpha using OLS regression for each stock/portfolio over the period. Then we measure the realised risk premia over the period:

$$\hat{\lambda}_k = \frac{1}{T} \sum_{t=1}^T F_{k,t}$$

for long-short factors or

$$\hat{\lambda}_k = \frac{1}{T} \sum_{t=1}^T F_{k,t} - r_f$$

for long-only factors.

and finally we decompose the ex-post average return:

$$\bar{R}_i = \alpha_i + \hat{B}_{i,market} \hat{\lambda}_{market} + \hat{B}_{i,size} \hat{\lambda}_{size} + \hat{B}_{i,value} \hat{\lambda}_{value} + \hat{B}_{i,mom} \hat{\lambda}_{mom} \quad (1.5)$$

The contribution of each factor to the performance of the stock or portfolio is given by the following formula:

$$PC(F_{k,i}) = \frac{\hat{B}_i^k \hat{\lambda}_k}{E(R_i)}$$

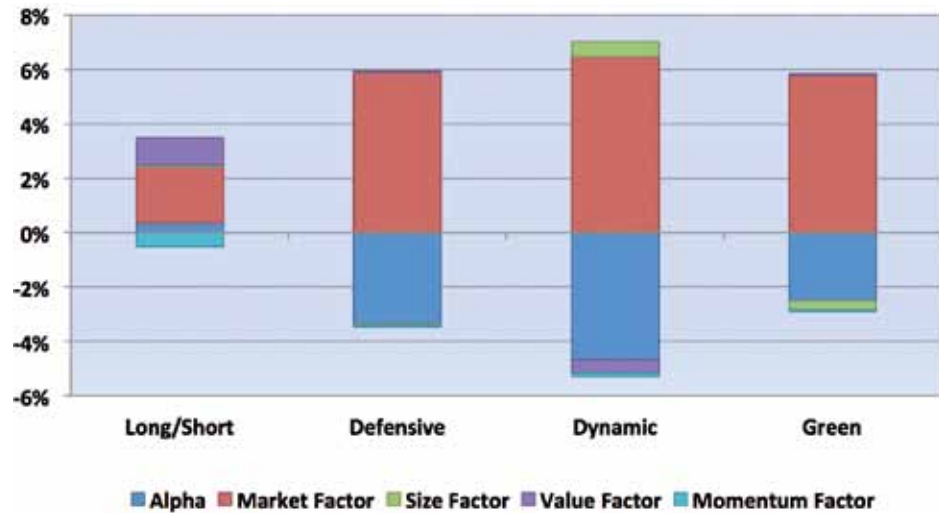
To provide an empirical illustration, we consider four US equity mutual funds from large providers, with different investment styles or themes: these funds are labelled by their respective providers as "long-short", "defensive", "dynamic" and "green". We regress monthly fund returns against the Carhart factors from Ken French's library, over the period 2001-2015. Figure 1 shows the decomposition of performance. Over the period considered, all ex-post risk premia are positive, with annual premia of 5.9% for the market factor, 3.8% for the size factor, 1.6% for the value factor and 2.3% for the momentum factor. Because the market premium is the largest and because long-only funds have market betas around one, the market factor explains most of the realised performance of these funds. It explains less for the long-short fund, whose market exposure is closer to zero. The abnormal performance, measured by the alpha in the regression (1.5), is actually negative.

After analysing the factors contribution to portfolio performance, we need to consider the risk that funds managers took to achieve those returns by considering factor contribution to overall risk portfolio.

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Figure 1: Performance Decomposition of Selected Mutual Funds with the Carhart Model (2001-2015)

Mutual fund returns are downloaded from Datastream and factor returns are from Ken French's library. All returns are monthly. We regress for each mutual fund their excess returns on factor returns for the period 2001-2015 and use ordinary least square (OLS) to estimate the factor exposures. We measure historical risk premia over the same period and use the formula (1.5).



## 1.2.2 Volatility Decomposition

Returns are linear functions of factors exposures making the expected return decomposition straightforward. Because the portfolio risk is non-linear in factor exposures, volatility decomposition is less immediate and we explore several methods to handle this issue. In what follows, we apply these methods to the decomposition of "absolute risk", that is volatility, and to "relative risk", defined as the tracking error with respect to a benchmark.

### 1.2.2.1 Absolute Versus Relative Risk

#### Volatility

We recall that multi-factor models for returns on portfolio have the general form:

$$R_t = B_1 F_{1,t} + B_2 F_{2,t} + \dots + B_k F_{k,t} + \varepsilon_t = \mathbf{B}' \mathbf{F}_t + \varepsilon_t$$

- $R_t$  is the return in excess of the risk-free rate on portfolio in time period  $t$  ( $t = 1, \dots, T$ ),
- $B_k$  is the factor loading or factor beta for the portfolio on the  $k$ -th factor, and  $\mathbf{B}$  is the vector of beta.

We make the following assumptions:

1. The factor realisations vector,  $\mathbf{F}_t$  is stationary with unconditional moments  $\text{cov}(\mathbf{F}_t) = E[(\mathbf{F}_t - E[\mathbf{F}_t])(\mathbf{F}_t - E[\mathbf{F}_t])'] = \Sigma_f$
2. Error terms,  $\varepsilon_t$  are uncorrelated with each of the common factors,  $F_{k,t}$   $\text{cov}(F_{k,t}, \varepsilon_t) = 0$  for all  $k$  and  $t$ .
3. Error terms  $\varepsilon_t$  are serially uncorrelated  $\text{cov}(\varepsilon_t, \varepsilon_s) = \sigma^2$  for  $t = s = 0$ , otherwise

With these assumptions, we can write the variance of the portfolio return as:

$$\sigma_p^2 = \mathbf{B}' \Sigma_f \mathbf{B} + \sigma^2 \quad (1.6)$$

With the variance fraction explained by the factor:  $\mathbf{B}' \Sigma_f \mathbf{B}$

The coefficient of determination from our model is the ratio of systematic variation to the total return variation:

$$R^2 = \frac{\mathbf{B}' \Sigma_f \mathbf{B}}{\sigma_p^2}$$

In the multi-factor framework, the systematic risk depends not only on the

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beta coefficients but also on the factors' variance-covariance matrix. Since the coefficient of determination is a measure of the systematic risk of portfolio, extracting the core, stand alone components of common factors enables us to decompose the systematic risk by disentangling the R-squared, based on factors' volatility and their corresponding betas.

In the presence of uncorrelated regressors (factors), the  $R^2$  of a multiple regression is the sum of the  $R^2$  of individual regressions, and the contribution of each factor to the variance of the portfolio is simply measured by the individual  $R^2$ . In Section 1.2.2.2 and Section 1.2.2.3, we present two methods for identifying within the explained component what the contribution of each factor is to the portfolio variance, in the more general case of correlated factors.

## Tracking Error

Portfolio risk is also frequently measured relative to a benchmark: the tracking error is defined as the standard deviation of the difference between the portfolio and benchmark returns.

In a return-based factor analysis, the methodology is similar to that employed for absolute risk decomposition, but it is the portfolio returns in excess of a benchmark that are regressed against long/short factor indices:

$$R_{p,t} - R_{B,t} = \bar{B}_1 F_{1,t} + \bar{B}_2 F_{2,t} + \dots + \bar{B}_k F_{k,t} + \varepsilon_t = \bar{\mathbf{B}}' \mathbf{F}_t + \varepsilon_t$$

- $R_{p,t} - R_{B,t}$  is the return on portfolio minus the return on benchmark index in time period  $t$  ( $t = 1, \dots, T$ )
- $\bar{B}_k$  is the factor loading or factor beta for the portfolio in excess of benchmark portfolio on the  $k$ -th factor,  $\bar{\mathbf{B}}$  is the vector of beta.

- $\varepsilon_t$  denotes the error term which is uncorrelated with each of the common factors and vector of error term is serially uncorrelated.

$$\sigma(R_p - R_B) = \sigma(\bar{\mathbf{B}}' \mathbf{F}_t + \varepsilon_t) = \sqrt{\bar{\mathbf{B}}' \boldsymbol{\Sigma}_f \bar{\mathbf{B}} + \sigma^2}$$

$\sigma(R_p - R_B)$  denotes the tracking error of the portfolio against its benchmark.

$$\sigma(R_p - R_B)^2 = \bar{\mathbf{B}}' \boldsymbol{\Sigma}_f \bar{\mathbf{B}} + \sigma^2 \quad (1.7)$$

The coefficient of determination from our model is the ratio of systematic variation to the total return variation:

$$R^2 = \frac{\bar{\mathbf{B}}' \boldsymbol{\Sigma}_f \bar{\mathbf{B}}}{\sigma(R_p - R_B)^2}$$

Thus, any method to decompose absolute risk can also be applied to the decomposition of relative risk, provided one considers the betas of returns in excess of the benchmark portfolio as opposed to the betas of returns in excess of the risk-free rate.

## *1.2.2.2 First Decomposition: Using Euler Decomposition of Volatility*

This approach is the one adopted in risk budgeting methodologies for constructing portfolios: the definition of the contributions of various assets to the volatility of a portfolio is based on a mathematical property of volatility, known as Euler decomposition (see Roncalli (2013), and also Qian (2006) for the economic interpretation of such risk decompositions). It is adapted here to a different context, where we are seeking to assess the contribution of different *factors* to the risk of a portfolio (the two main differences being that the betas do not sum up to one, and that there is a residual term in factor analysis, which is not present when decomposing a portfolio return into the weighted sum of the components' returns).

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We consider a set of assets with a covariance matrix  $\Sigma$ . Let  $R(\mathbf{x})$  be a risk measure of the portfolio  $\mathbf{x} = (x_1, \dots, x_n)$ . If this risk measure is homogenous of degree 1 in the weights, it satisfies the Euler principle, and we have

$$RC(\mathbf{x}) = \sum_{i=1}^n x_i \frac{\partial R(\mathbf{x})}{\partial x_i}.$$

Roncalli and Weisang (2012) define the risk contribution  $RC_i$  of asset  $i$  as the product of the weight by the marginal risk:

$$RC_i = x_i \frac{\delta R(\mathbf{x})}{\delta x_i}.$$

If we use the volatility of the portfolio  $\sigma(\mathbf{x}) = \sqrt{\mathbf{x}'\Sigma\mathbf{x}}$  as the risk measure, it follows that the contribution of the asset  $i$  to the portfolio volatility is:

$$RC_i = x_i \frac{(\Sigma \mathbf{x})_i}{\sqrt{\mathbf{x}'\Sigma\mathbf{x}}}.$$

Roncalli and Weisang (2012) define also the risk contribution with respect to the factors. Here, we focus on the contribution of the factor  $k$  to the systematic variation of portfolio (the fraction of variance explained by the factors). With the previous notations, we have the following risk contribution of factor  $k$ :

$$RC(F_k) = B_k \frac{\partial R(\mathbf{B})}{\partial B_k},$$

where the risk measure is the systematic variance of portfolio returns:  $R(\mathbf{B}) = \mathbf{B}'\Sigma_f\mathbf{B}$ .

In the presence of uncorrelated factors, there is no correlated component in the factors' variance-covariance matrix and we have the following risk contribution of factor  $k$ :

$$RC(F_k) = \frac{B_k^2 \times \sigma F_k^2}{\mathbf{B}'\Sigma_f\mathbf{B}}$$

with

$$\sum_{k=1}^K RC(F_k) = \sum_{k=1}^K \frac{B_k^2 \times \sigma F_k^2}{\mathbf{B}'\Sigma_f\mathbf{B}} = 1.$$

The problem of assigning correlated components is addressed by attributing half of each correlated component to each one of the two factors. In developed form, the risk contribution of factor  $k$  is:

$$RC(F_k) = B_k \frac{(\Sigma_f \mathbf{B})_k}{\mathbf{B}'\Sigma_f\mathbf{B}} \quad (1.8)$$

In this approach, the factors' covariance matrix  $\Sigma_f$  and the factors' loading vector  $\mathbf{B}$  used to compute risk contribution of each factor are not observable. We need to estimate covariances between factors, generating a covariance matrix estimate  $\widehat{\Sigma}_f$ . The simplest estimate is the sample covariance matrix, but we can also apply an exponentially decreasing weighting scheme to historical observations; it allows placing more weight on recent observations. We estimate the factors loading vector  $\widehat{\mathbf{B}}$  by regressing portfolio returns on factor returns (ordinary least squares).

Bhansali et al. (2012) point the benefits of using a factor-based diversification measure with respect to asset-based measures. Using a principal component analysis, they extract two risk factors driven by global growth and global inflation, and argue that these two risk factors dominate asset class risk and return. They extract the risk factors from a sample universe of 9 conventional assets: U.S. equities, International equities, EM equities, REITS, commodities, global bonds, U.S. long Treasury, investment grade corporate bonds and high yield bonds. Then, they decompose the return and the variance of an asset into the following two factors: the growth risk (linked with equity risk), the inflation risk (linked with bond risk) and a residual risk that is not spanned by equity and bond factors. For the variance decomposition, they equally divide the covariance term between the bond and the equity components. The above two-factor variance decomposition

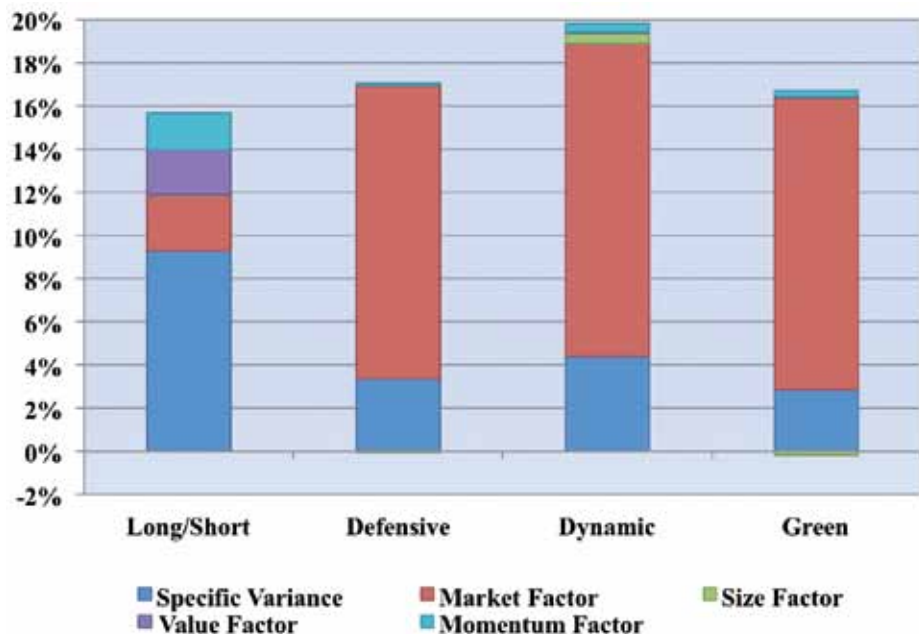
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allows them to quantitatively examine the returns of any risk parity strategy to determine whether their growth (equity) and inflation (bond) risk are indeed in parity. They consider a portfolio made of the S&P 500 index and the 10-year Treasury US bond, and they find that achieving parity in risk exposures has a positive impact on the Sharpe ratio, especially during bear period. They also show that asset-based risk parity portfolios can often concentrate too much in just one component of risk exposures, particularly equity risk, in contrast to factor-based risk parity which allows a more robust risk diversification.

To illustrate this decomposition, we consider the same mutual funds as in Section 1.2.1, and we plot the contributions of the Carhart factors to their volatility in Figure 2. Most of the ex-post risk of the long-only funds in the Figure 2 comes from their market exposure and from specific risk. This specific risk was not rewarded in

this sample, as appears from the negative alphas in Figure 1, while market risk was well rewarded over the period. The long/short fund has the highest specific variance and the lowest market risk but this specific risk is not rewarded for this fund over the period while the market factor obtains the highest reward. Common intuition and portfolio theory both suggest that the degree of diversification of a portfolio is a key driver of its ability to generate attractive risk-adjusted performance across various market conditions. Carli, Deguest and Martellini (2014) argue that balanced factor contributions to portfolio risk lead to higher performance in the long run (due to higher performance during bear market). Hence, fund managers could manage a better risk factor diversification with more balanced factor exposures. For this aim, they need to analyse and measure with accuracy their fund risk factor exposures.

Figure 2: Risk (Volatility) Decomposition Using Euler Decomposition of Selected Mutual Funds with the Carhart Model (2001-2015) Mutual fund returns are downloaded from Datastream and factor returns are from Ken French's library. Returns are monthly. We regress for each mutual fund their excess returns on factor returns for the period 2001-2015. We measure historical factor covariances matrix over the same period and use the formula (1.8).

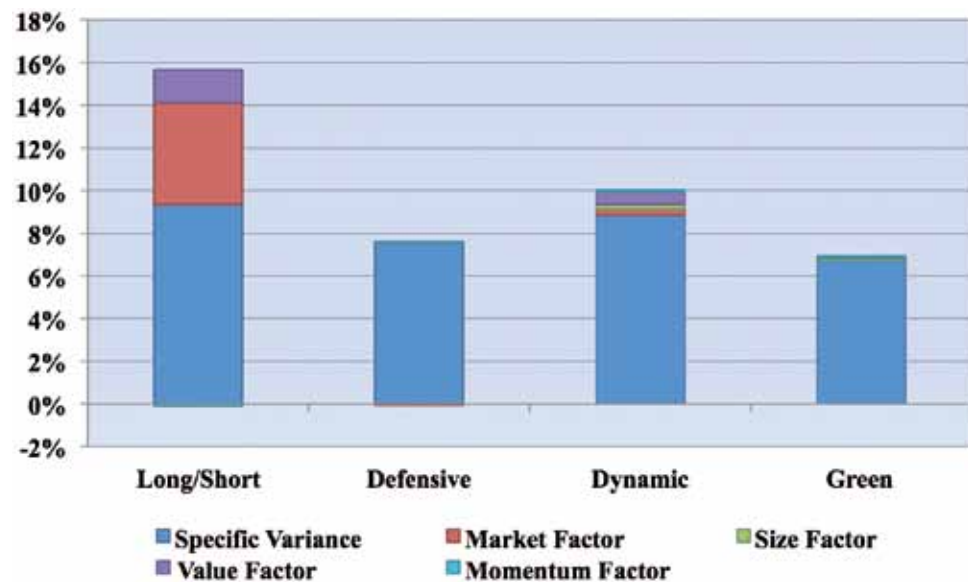




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Figure 3: Tracking Error Decomposition of Selected Mutual Funds with the Carhart Model (2001-2015)

Mutual fund returns are downloaded from Datastream and factor returns are from Ken French's library. Returns are monthly. We regress for each mutual fund their returns in excess of the market return on factor returns for the period 2001-2015. Market returns are from Ken French's library. We measure historical factor covariances matrix over the same period and use the formula (1.7) and (1.8).



In Figure 3, we apply the same method to the decomposition of tracking error with respect to the market factor. By subtracting market returns from the returns to long-only fund, the influence of the market factor is largely reduced, so the market factor explains much less of the tracking error than of the volatility of long-only funds. For the long-short fund, it is the opposite: the market factor has little impact on the fund's returns, so taking excess returns reinforces this impact.

### 1.2.2.3 Second Decomposition: Orthogonalising Factors

The problem of attributing correlated components in the expression of volatility or tracking error would be avoided if factors were orthogonal. Hence, a second idea to perform a risk decomposition is to transform the original factors into uncorrelated factors by using some rotation technique. The new factors are linear combinations of the original ones, so they generate the same set of uncertainty and they explain exactly

the same fraction of the returns to a given portfolio: the coefficient of determination (R-squared, i.e. the ratio of systematic variance to overall variance of the portfolio) is the same with the orthogonal factors than with the original ones. This approach "hides" the arbitrary decision of how to assign the correlated component with the somewhat arbitrary selection of an orthogonalisation methodology.

There exist several alternative linear transformations  $\tilde{\mathbf{F}} = \mathbf{tF}$ , or torsions, of the original factors, that are uncorrelated, and that are represented by a suitable  $m \times m$  de-correlating torsion matrix  $\mathbf{t}$ . One natural way to turn correlated asset returns into uncorrelated factor returns is to use principal component analysis (PCA). While useful in other contexts, the PCA approach suffers from a number of shortcomings when computing the factor relative risk contribution (Carli, Deguest and Martellini, 2014). The first shortcoming is the difficulty in interpreting the factors, which are pure

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statistical artefacts. The second shortcoming, particularly severe in the context of the design of a relative contribution measure, is that by construction, principal components are defined so as to achieve the highest possible explanatory power. As a result, the contribution of the first few factors is often overwhelmingly large with respect to the contribution of other factors, and the contribution of the remaining factors tends to be biased towards low values. The minimum linear torsion technique of Deguest, Martellini and Meucci (2013) avoids these problems and facilitates the interpretation of the orthogonalised factors. The minimum-torsion factors  $\tilde{F}$  are defined as the factors that have the same variances as the original ones and minimise the sum of tracking errors with respect to them. Mathematically, they solve the program:

$$\min \sum_{k=1}^K \sigma(\tilde{F}_k - F_k)^2,$$

$$\text{subject to } \sigma(\tilde{F}_k) = \sigma(F_k)$$

for all  $k = 1, \dots, K$ .

An explicit expression for the minimum-torsion matrix  $\mathbf{t}$  can be found in Carli, Deguest and Martellini (2014).

Once factors have been made orthogonal, we estimate the exposures  $\tilde{B}_k$  of a portfolio with respect to the uncorrelated factors, as well as the idiosyncratic return  $\eta_t$ :

$$R_t = \sum_{k=1}^K \tilde{B}_k \tilde{F}_{k,t} + \eta_t = \tilde{B} \tilde{F}_t + \eta_t.$$

The portfolio variance now involves no covariance term:

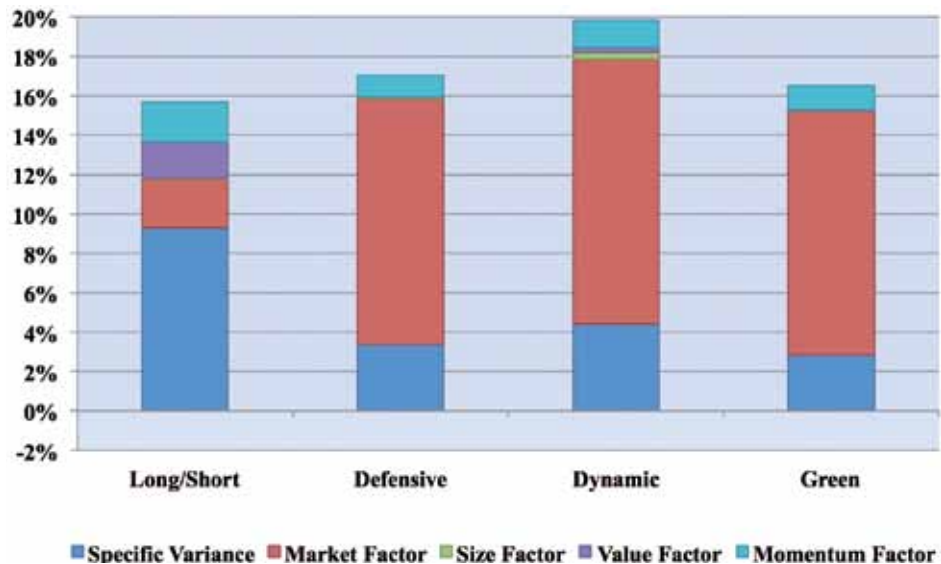
$$\sigma_p^2 = \sum_{k=1}^K \tilde{B}_k^2 \sigma(F_k)^2 + \sigma_\eta^2,$$

so the contribution of each factor is:

$$RC(\tilde{F}_k) = \frac{\tilde{B}_k^2 \sigma(F_k)^2}{\sum_{i=1}^K \tilde{B}_i^2 \sigma(F_i)^2} = \frac{\tilde{B}_k^2 \sigma(F_k)^2}{\sigma_p^2 - \sigma_\eta^2}. \tag{1.9}$$

Figure 4: Risk (Volatility) Decomposition with Orthogonalised Factors of Selected Mutual Funds with the Fama-French 4 Factor Model (2001-2015)

Mutual fund returns are downloaded from Datastream and factor returns are from Ken French's library. Returns are monthly. We first orthogonalise the risk factors via the minimum linear torsion approach. We then regress for each mutual fund their excess returns against the orthogonalised factor returns for the period 2001-2015. The minimum-torsion factors have the same variance as the original factors. We measure historical factor covariance matrix over the same period and use Equation (1.9) to perform the risk attribution.





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As in the previous methods, we first estimate the factors' covariance matrix  $\widehat{\Sigma}_f$  and the beta vector  $\widehat{B}$ , and we then solve the minimum-torsion optimisation so as to obtain the minimal torsion transformation  $t$ . Next, we perform the variance decomposition according to Equation (1.9).

Figure 4 shows the result of this procedure for the four mutual funds. It is very close to Figure 2, because the long-short factors have low correlations between themselves and with the market, so that making them perfectly orthogonal has only a small impact on the results. Most of the ex-post risk of the long/only funds in Figure 4 comes from their market exposure and from specific risk. The long/short fund has the highest specific variance and the lowest market risk. The only change comes from the momentum factor, which has a higher risk contribution with the MLT method because of its negative correlation with the market factor over the period.

## 1.2.2.4 Other Methods for Variance Decomposition

Several other methods can be considered to decompose the risk of a portfolio. This subsection presents two of them, which can be applied to volatility or to tracking error.

### Disregarding Correlated Components

Another approach consists in keeping the previous marginal contribution of factor definition but we overlook the correlated component.

$$RC(F_k) = \frac{B_k^2 \times \sigma(F_k)^2}{B' \Sigma_f B}$$

Hence, we do not attribute the correlated components and leave them together as a separate contributor to portfolio risk.<sup>6</sup>

In this case, long-short and long-only factors should be differentiated. Long-only factors have a higher correlation than long-short factors, which implies a higher correlated component. Hence this approach is similar to the previous one for the risk attribution if long-short indices are used as investment vehicles for the factors.

As for the first approach we need to compute factors' covariance matrix estimate  $\widehat{\Sigma}_f$  and estimate the factor loading vector  $\widehat{B}$ .

### Decomposition of R-squared

The last approach deals with assessing the relative explanation of regressors in linear regressions based on the portfolio variance decomposition. The key difficulty is to decompose the total variance or R-squared, when regressors (here, risk factors) are correlated. This problem has been discussed in the statistical literature on the relative importance of correlated regressors in multivariate regression models (Chevan and Sutherland, 1991). The idea is to take the average over all possible permutations of the marginal increase in R-squared related to the introduction of a new regressor starting from a given set of existing regressors.

$R2(1:K)$  denotes the coefficient of determination of the linear regression with the K factors as regressors and  $R2(k)$  denotes the coefficient of determination of the linear regression with only the  $k^{\text{th}}$  factor. When the factors are pairwise uncorrelated, i.e.  $\rho(F_k, F_l) = 0$  for  $k \neq l$ , we have:

$$R2(1:K) = \sum_{k=1}^K R2(k)$$

This is the decomposition of R-squared in a multivariate model with orthogonal regressors.

<sup>6</sup> -This approach is similar to the approach developed in Deguest, Martellini and Milhau (2012), where the focus was on the contribution of various assets to investors' welfare, and where the correlated components were grouped together in a term that was called "diversification component".

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We can define the relative contribution of one risk factor to the risk of the portfolio as the fraction of variance explained by the factor in the linear regression:

$$RC(F_k) = \frac{R2(k)}{R2(1:K)}$$

To see how to build a measure of relative importance, we first consider a simple example with two risk factors. There are two ways of decomposing  $R2(1,2)$ :

$$R2(1,2) = R2(1) + R2(1,2|1)$$

and

$$R2(1,2) = R2(2) + R2(2,1|2)$$

In the two decompositions above, one clearly sees that the order of introduction of a given factor matters in its contribution to the total coefficient of determination. In other words, the coefficient of determination of adding factor 2 to factor 1, denoted  $R2(1, 2|1)$ , is different from the coefficient of determination of only factor 2, denoted  $R2(2)$ , that is unless the two factors are uncorrelated. One way to get rid of this dependency consists in decomposing the  $R2$  as:

$$\begin{aligned} R2(1,2) &= \frac{1}{2} [R2(1) + R2(1,2|1) + R2(2) + R2(2,1|2)] \\ &= \frac{1}{2} [R2(1) + R2(1,2|1)] + \frac{1}{2} [R2(2) + R2(2,1|2)] \end{aligned}$$

In this context, one may define the relative importance of factor 1 and 2 as the average contribution of the factor under consideration over all possible permutations of the set of existing factor, or, in this example:

$$RC(F_1) = \frac{R2(1) + R2(2,1|2)}{2 \times R2(1,2)}$$

And

$$RC(F_2) = \frac{R2(2) + R2(1,2|1)}{2 \times R2(1,2)}$$

If  $F_1$  and  $F_2$  are orthogonal, we verify:

$$RC(F_1) + RC(F_2) = \frac{R2(1)}{2 \times R2(1,2)} + \frac{R2(2)}{2 \times R2(1,2)} = 1$$

This example with  $K = 2$  can be generalised. For  $j \geq 1$ , we let  $\zeta_j(i)$  denote the set of permutations of  $j$  factors chosen among  $1, \dots, K$  and distinct of  $i$  (no repetition is allowed, and the order matters), and  $\zeta_0(i)$  denotes the singleton that contains only the empty set.

Assume that  $R2(1:K)$  is not zero, we define the relative risk contribution of factor  $k$  as:

$$RC(F_k) = \frac{\sum_{i=0}^{K-1} (K-i-1)! \sum_{\zeta \in \zeta_j(i)} R2(\zeta, k|\zeta)}{K! \times R2(1:K)}$$

Then we have that

- 1) The relative risk contributions are nonnegative;
- 2) The relative risk contributions of the various factor sums up to 1:  $\sum_{k=1}^K RC(F_k) = 1$
- 3) If factor  $1, \dots, K$  are orthogonal to each other, then the relative contribution of factor  $k$  is given by:

$$RC(F_k) = \frac{R2(k)}{R2(1:K)}$$

In the aim to access factor relative contributions, with a four-factor model, we would need to run 8 regressions for each factor. Hence it would give us 32 regressions to perform. For each regression we would obtain the observable marginal increase of the coefficient of determination by adding the new factor needed to compute relative risk contribution of each factor.

## 1.3 Other Factor Models Used in Practice

The Fama-French and the Carhart models are at the intersection between academic and industry practices. They are commonly

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used by academics and practitioners to attribute the performance of a fund to systematic risk factors or to the manager's skills. But other models have been developed by market practitioners to include other factors which are deemed important to explain the risk and the performance of a fund. The most famous example is the family of Barra models, which we present in Section 1.3.1. In Section 1.3.2 and Section 1.3.3, we take a closer look at the problem of multi-collinearity in models with a large number of factors and we present two possible methods to address this concern.

## 1.3.1 Barra Model

There are in fact several classes of Barra models, which differ through the set of factors employed. One important difference between these models and the empirical asset pricing models of Fama and French or Carhart is that they treat factor exposures as observed quantities, as opposed to parameters to be estimated from a regression. It is the factors that are regarded as unobservable variables. The factor values at each date  $t$  are estimated through a cross-section regression of stock returns on the predetermined betas. Hence, the Barra method is an implicit factor method which involves extracting the factors directly from the historical returns.

Mathematically, the excess return of a stock  $i$  between dates  $t - 1$  and  $t$  is expressed as:

$$R_{i,t} = \sum_{c=1}^C B_{i,t,c}^{(C)} F_{c,t} + \sum_{s=1}^S B_{i,t,s}^{(S)} F_{s,t} + \sum_{r=1}^R B_{i,t,r}^{(R)} F_{r,t} + \varepsilon_{i,t} \quad (1.10)$$

with

$$\mathbf{B}_{i,t}^{(C)} = \left( \begin{array}{c} B_{i,t,1}^{(C)} \\ \dots \\ \dots \\ B_{i,t,C}^{(C)} \end{array} \right) \Bigg\} C,$$

$$\mathbf{B}_{i,t}^{(S)} = \left( \begin{array}{c} B_{i,t,1}^{(S)} \\ \dots \\ \dots \\ B_{i,t,S}^{(S)} \end{array} \right) \Bigg\} S,$$

$$\mathbf{B}_{i,t}^{(R)} = \left( \begin{array}{c} B_{i,t,1}^{(R)} \\ \dots \\ \dots \\ B_{i,t,R}^{(R)} \end{array} \right) \Bigg\} R,$$

Here, the set of factors has been split into three subsets:  $C$  country factors,  $S$  sector factors and  $R$  risk factors.  $B_{i,t,c}^{(C)}$  denotes a dummy variable and takes the value one if asset  $i$  belongs to country  $c$  or 0 otherwise, and  $B_{i,t,s}^{(S)}$  is also a dummy variable and takes the value one if asset  $i$  belongs to sector  $s$  or 0 if not. The risk index exposures  $B_{i,t,r}^{(R)}$  are defined as continuous variables and are normalised within the country by ranking the company in each factor relatively to other local companies. The factor values at date  $t$ ,  $\widehat{F}_{c,t}$ ,  $\widehat{F}_{s,t}$  and  $\widehat{F}_{r,t}$  are estimated by performing the cross-section regression (1.10) at date  $t$ .

This methodology is employed in the BARRA Global Equity Model (see the Risk Model Handbook (1998)). The Global Equity Model is a multi-factor model, partitioned into specific return and common factor return, whose main purpose is to assess the relative contributions of industry versus country factors. The common factors are industries, countries and risk indices. The equation for the Global Equity Model is Equation (1.10), with 90 factors in the MSCI version, and 93 factors in the FT version (1998).

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$B_{i,t,c}^{(C)}$ ,  $B_{i,t,s}^{(S)}$  and  $B_{i,t,r}^{(R)}$  in Equation (1.10) are predetermined exposures describing the relevant asset characteristics: local market, industry and risk index exposures. The Global Equity Model assigns assets to industry categories by mapping industry data to MSCI or FT classifications. GEM assigns each security to a single industry; hence industry factor exposures are 0 or 1 for each asset. Industry risk exposures indicate the percentage of total portfolio value in each industry classification. They use the same methodology for country factor exposures; GEM assigns each security to a single country, so that country exposures are also 0 or 1 for each asset. The risk indices used are the size index, the success (momentum) index, the value index and the variability in market index (volatility historical index). Risk index are defined as z-scores of companies within companies of the same country. For example, a firm whose size would be the average size of all companies would have a zero exposure to the size factor. The Global Equity Model allows to compare local country factor returns, local risk index returns, local industry returns across countries and to access the contribution of each factor relative to the others. For instance, Grinold, Rudd and Stefek (1989) examine the statistical significance of each factor's return or the absolute level of volatility of each factor across countries.

If a stock is quoted in another currency than the dollar, the decomposition in Equation (1.10) is applied to the local currency returns, and the dollar return is expressed as the sum of the local return and a currency return. Figure 5 summarises the decomposition process.

Menchero and Morozov (2011) investigate the relative importance of countries and industries across the entire world by constructing a global factor model containing one world factor, 48 country factors, 24 industry factors, and 8 risk index factors. Following the Barra Global Equity Model (1998), they assign country exposures and industry exposures thanks to MSCI classification and every stock is assigned an exposure of 1 to the world factor. The 8 risk indexes factors are volatility (essentially representing market beta), size, momentum, value, growth, leverage, liquidity, and non-linear size. The authors use local excess returns in all regressions; this eliminates currency effects and allows them to have a common basis to compare countries and industries.

### 1.3.2 Extending a Set of Pricing Factors with Ad-Hoc Factors

While Barra model treats factor exposures as predetermined quantities, it is also possible to add new factors to a set of pricing factors and to estimate the

Figure 5: Excess Return Decomposition with the Global Equity Model



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exposures to the extended model by linear regression. Introducing new explicit factors, however, creates concerns in terms of multi-collinearity: if some combination of the added factors is too correlated with a combination of the original ones, then exposures cannot be reliably estimated. As a solution to this problem, Menchero and Poduri (2008) propose to make the new factors orthogonal to the existing ones.

## The Blended Model

We use the same notation as Menchero and Poduri (2008), albeit with a slightly different meaning. The core factor model consists of a set of  $K$  factors  $F^{(X)}$ , which we interpret here as common sources of risk and performance. Thus, we think of  $F^{(X)}$  as the Fama-French or Carhart factors for example, while the new factors  $F^{(Y)}$  are  $L$  sector or country factors. In Menchero and Poduri's work,  $F^{(X)}$  represents the data-generating process for stock returns (a Barra model) and the "custom factors"  $F^{(Y)}$  are the Fama-French factors. These differences in interpretation of the notation have no effect on the methodology. The blended model contains all factors, which can be numerous. For instance, standard sector classifications such as MSCI's Global Industry Classification Standard, FTSE's Industry Classification Benchmark and Thomson Reuter Business Classification contain ten sectors at the broadest level, and this number grows as one moves to finer decomposition levels. With so many regressors, a direct linear regression of portfolio returns may be unable to disentangle the respective effects of the various factors.

Menchero and Poduri's approach is to regress the factors  $F^{(Y)}$  on the factors  $F^{(X)}$ , and take the residuals  $F^{(Y)\perp}$  so as to obtain the effects of factors  $Y$  net of factors  $X$ . Next, portfolio returns  $R_{p,t}$  are regressed against the whole set of factors:

$$R_{p,t} = \sum_{l=1}^K B_{i,l} F_{i,t}^{(X)} + \sum_{k=1}^L B_{i,k} F_{k,t}^{(Y)\perp} + \varepsilon_t.$$

Note that this approach is return-based, meaning that it requires neither knowledge of the portfolio holdings nor of any fundamental or market information at the stock level. As such, it is less demanding in terms of inputs than a Barra-type model.

## Sector and Carhart Orthogonalised Factors

We illustrate this method by taking the market as the single core factor because it is the first-order source of risk in returns, and consider the "custom" factors to be the sector and the Carhart factors. We consider the US equally-weighted broad index over the period 2002–2015. We first regress equally-weighted sector returns  $R_s$  on the market factor, and we collect the residual plus the intercept term to isolate the "pure sector effect"  $v_s$ ,

$$R_{s,t} = \alpha_s + \beta_s R_{m,t} + \eta_{s,t},$$

$$v_{s,t} = \alpha_s + \eta_{s,t}.$$

We do the same to extract the net effect of Carhart factors. Let  $R_k$  be the return on one of the four Fama-French-Carhart portfolios and  $v_k$  be the corresponding factor purged of the market influence:

$$R_{k,t} = \alpha_k + \beta_k R_{m,t} + \eta_{k,t},$$

$$v_{k,t} = \alpha_k + \eta_{k,t}.$$

Finally, we regress US equally-weighted broad index returns  $R_p$  on market factor augmented by the pure sector effect and the pure pricing factor effect:

$$R_{p,t} = \alpha_p + B_p^m R_{m,t} + \sum_{s=1}^S B_p^s v_{s,t} + \sum_{k=1}^3 B_p^k v_{k,t} + \varepsilon_{p,t}.$$

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We then proceed to performance and risk decomposition. Taking sample averages in both sides of the previous equation, we obtain, for a portfolio (or an individual stock)  $p$ :

$$\bar{R}_p = \hat{\alpha}_p + \hat{B}_p^m \hat{\lambda}_m + \sum_{s=1}^S \hat{B}_p^s \hat{\alpha}_s + \sum_{k=1}^3 \hat{B}_p^k \hat{\alpha}_k \quad (1.11)$$

By construction, residuals are orthogonal from factors, so the portfolio variance can be written as:

$$\hat{\sigma}_p^2 = \hat{B}' \hat{\Sigma}_f \hat{B} + \hat{\sigma}_\varepsilon^2.$$

We decompose the systematic portfolio risk using the Euler decomposition of volatility (see Section 1.2.2.2).  $\hat{\Sigma}_f$  denotes the sample covariance matrix of all factors (the market plus the sectors and the additional pricing factors) and we have the following risk contribution of factor  $k$  to the portfolio risk:

$$RC(F_k) = \hat{B}_k \frac{(\hat{\Sigma}_f \hat{B})_k}{\hat{B}' \hat{\Sigma}_f \hat{B}}. \quad (1.12)$$

These decompositions hold ex-post, but similar expressions can be written down for ex-ante expected return and variance. They can also be applied by replacing portfolio returns with returns in excess over a benchmark in an attempt to provide a statistical explanation for the outperformance or the underperformance of the portfolio with respect to the benchmark and for the tracking error.

This method alleviates the multi-collinearity concern. By eliminating the market from the additional factors (Carhart factors and sectors), it drastically reduces their level of pairwise correlation. Collinearity is less an issue for the Carhart factors, which are already long-short returns with moderate correlations, than for the sectors, which are long-only returns. Much of the correlation between the sectors arises indeed because

of their common loadings on the market factor, and eliminating the effect of this factor reduces their correlations. To give a quantitative sense of this effect, the average correlation across the 10 long-only sector returns is 0.72, while it is only 0.37 between the pure sector returns. This reduction in the correlation levels enables us to disentangle the effects of sectors on the portfolio more reliably (i.e. with larger t-statistics).

Finally, the method can be straightforwardly extended to include other custom factors such as country benchmarks, in order to capture the effects of geography on performance and risk. To this end, we regress equally-weighted country portfolios returns  $R_c$  on the market factor returns and we form the pure country effect  $v_c$ .

$$R_{c,t} = \alpha_c + \beta_c R_{m,t} + \eta_{c,t}$$

$$v_{c,t} = \alpha_c + \eta_{c,t}$$

The more comprehensive model, with all pricing factors, sector and country effects, reads:

$$R_{p,t} = \alpha_p + B_p^m R_{m,t} + \sum_{s=1}^S B_p^s v_{s,t}$$

$$+ \sum_{k=1}^3 B_p^k v_{k,t} + \sum_{c=1}^C B_p^c v_{c,t} + \varepsilon_{p,t}$$

## Empirical Illustration

We illustrate this method by taking as a test portfolio the equally-weighted portfolio of the S&P 500 universe over the period 2002–2015 and the market factor from Ken French's library. The analysis is conducted with quarterly returns, and the universe's constituents are classified into ten sectors according to the TRBC classification: Energy, Basic Materials, Industrials, Cyclical Consumer, Non-Cyclical Consumer, Financials, Healthcare, Technology, Telecoms and Utilities. We define sector returns as the returns to equally-weighted portfolios



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of each sector. In what follows, we apply the previous method to the decomposition of "absolute" risk and performance that are respectively volatility and historical return, and to "relative" risk and performance, defined respectively as the tracking error and the excess return with respect to the market factor.

Figure 6 shows the relative and absolute performance decomposition in the one-factor model augmented with pure sector effects and pure factor effects. Although the number of explanatory variables is greater than in the Carhart model, the market factor remains largely dominant to explain the realised performance, as it was for the long-only mutual funds in Section 1.2.1. It is only for relative performance that sector effects play a more significant role than the market: their contributions add up to 3.10%, versus only 0.10% for factor effects.

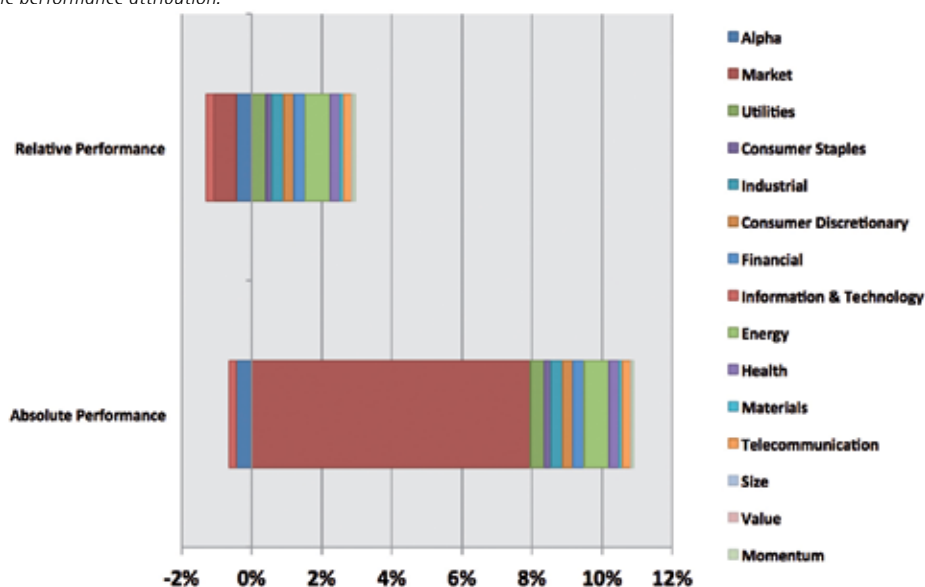
Finally, alpha has a negative contribution in both cases.

We perform the absolute and relative risk decomposition in Figure 7. Most of ex-post volatility is due to market risk, while pure sector effects are dominant in relative risk. By using more factors, the blended model has lower alpha and specific risk than the Carhart model.

As a general comment, this method is easy to apply, and, as noted before, it requires minimal information on the portfolio that is decomposed since it only takes the returns as inputs (return-based approach). On the other hand, if and when available, holding-based information could serve as a useful prior and should not be discarded. In the next section, we precisely introduce a method that takes advantage of knowledge of the portfolio composition.

Figure 6: Absolute and Relative Performance Decomposition of the Equally-Weighted Portfolio of the S&P 500 Universe on Market, Carhart and Sector Factors

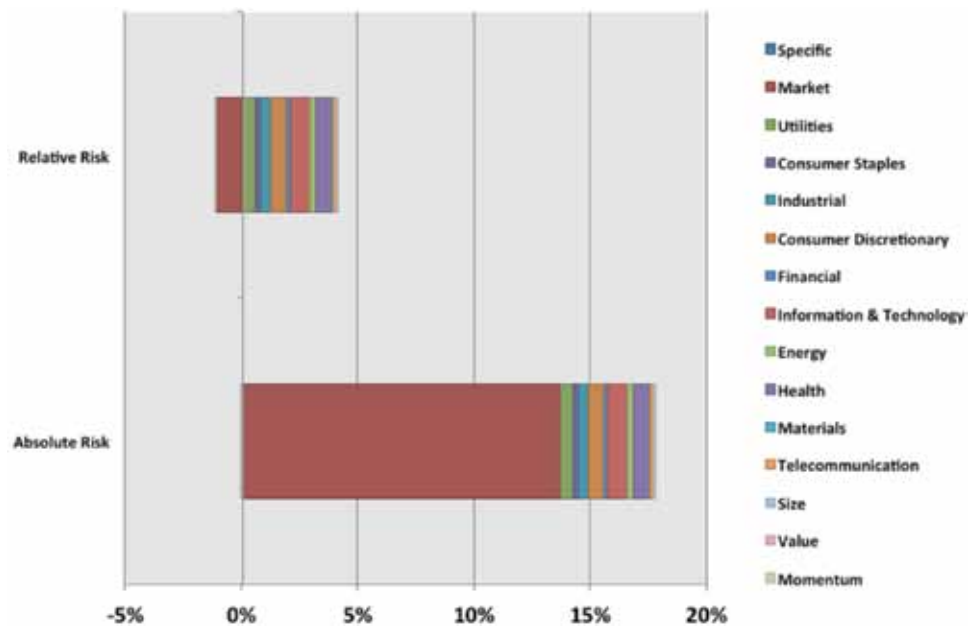
Factor returns are from Ken French's library. Sector returns are equally-weighted portfolios from the S&P 500 universe. We regress quarterly excess returns to each sector and factor portfolio on the market factor returns over the period 2002-2015 in order to extract pure sector and factor effects. We then regress equally-weighted portfolio excess returns on the market factor returns augmented with the pure sectors and factor effects, so as to obtain the factor exposures. For absolute performance, equally-weighted portfolio returns are in excess of the risk-free rate; for relative performance, they are in excess of market returns. We use formula (1.11) to make the performance attribution.



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Figure 7: Absolute and Relative Risk (Volatility) Decomposition Using Euler Decomposition for the Equally-Weighted Portfolio of the S&P 500 Universe on Market, Carhart and Sector Factors

Factor returns are from Ken French's library. Sector returns are equally-weighted portfolios from the S&P 500 universe. We regress quarterly excess returns to each sector and factor portfolio on the market factor returns over the period 2002-2015 in order to extract pure sector and factor effects. We then regress equally-weighted portfolio excess returns on the market factor returns augmented with the pure exposures and factor effects, so as to obtain the factor exposures. For absolute performance, equally-weighted portfolio returns are in excess of the risk-free rate; for relative performance, they are in excess of market returns. We use formula (1.12) to make the risk attribution.



### 1.3.3 Using the Sector Composition

In this approach, a portfolio is viewed as a bundle of sector portfolios, which are themselves projected on risk factors. In this sense, the method is both holding-based (it relies on the expression of the portfolio return as the weighted sum of constituents' return) and return-based (the sector returns are regressed against the factors).

Portfolio weights are usually time-varying, due to rebalancing and price changes. Hence, we now decompose the ex-ante return and volatility conditional on the weights of a given date, as opposed to writing an ex-post decomposition for the realised return and volatility over a period. Thus, the decomposition does not apply to past performance and risk but instead to the expected return and risk, estimated in a forward-looking way.

The starting point is the relation between the return to a portfolio  $p$  and those of its  $N$  constituents. Stocks can be grouped in the  $S$  sectors in this expression:

$$R_{p,t} = \sum_{i=1}^N w_{i,t-1} R_{i,t} = \sum_{i=1}^N \sum_{j=1}^S w_{i,t-1} R_{i,t} s_{i,t}^j$$

where  $w_{i,t-1}$  denotes the weight of asset  $i$  in portfolio  $p$  at period  $t-1$ ,  $r_{i,t}$  the return on stock  $i$  at period  $t$ , and  $s_{i,t}^j$  is a dummy variable equal to 1 if stock  $i$  belongs to sector  $j$  at period  $t$ , and 0 otherwise. We then define  $w_{t-1}^j$  as the  $j$ -th sector weight in portfolio  $p$  and  $\tilde{w}_{i,t-1}^j$  as the weight of asset  $i$  within sector  $j$ :

$$w_{t-1}^j = \sum_{i=1}^N w_{i,t-1} s_{i,t}^j$$

$$\tilde{w}_{i,t-1}^j = \frac{w_{i,t-1}}{w_{t-1}^j}$$



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For an equally- or cap-weighted portfolio, the sector portfolios are themselves equally- or cap-weighted. This allows us to express the return on sector portfolio  $j$  as:

$$r_t^j = \sum_{i=1}^N \tilde{w}_{i,t-1}^j R_{i,t} s_{i,t}^j$$

We can now rewrite the return on portfolio  $p$  as a linear combination of returns on sector portfolios:

$$\begin{aligned} R_{p,t} &= \sum_{i=1}^N \sum_{j=1}^S w_{i,t-1} R_{i,t} s_{i,t}^j = \sum_{i=1}^N \sum_{j=1}^S w_{t-1}^j \tilde{w}_{i,t-1}^j R_{i,t} s_{i,t}^j \\ &= \sum_{j=1}^S w_{t-1}^j \sum_{i=1}^N w_{i,t-1}^j R_{i,t} s_{i,t}^j = \sum_{j=1}^S w_{t-1}^j r_t^j \end{aligned}$$

The next step is to decompose the returns on sector portfolios according to the Carhart model:

$$\begin{aligned} r_t^j &= \alpha_j + B_{j,market} (Rm_t) + B_{j,size} (SMB_t) \\ &\quad + B_{j,value} (HML_t) + B_{j,mom} (WML_t) + \varepsilon_{j,t} \end{aligned}$$

Substituting this expression for each sector in the expression for the portfolio returns:

$$\begin{aligned} R_{p,t} &= \sum_{j=1}^S w_{t-1}^j r_t^j \\ &= \sum_{j=1}^S w_{t-1}^j [\alpha_j + B_{j,market} (Rm_t) + B_{j,size} (SMB_t) \\ &\quad + B_{j,value} (HML_t) + B_{j,mom} (WML_t) + \varepsilon_{j,t}] \end{aligned}$$

By identifying factor loadings we obtain the following expressions for the portfolio alpha and beta:

$$\begin{aligned} \alpha_p &= \sum_{j=1}^S w_{t-1}^j \alpha_j \\ B_{p,k} &= \sum_{j=1}^S w_{t-1}^j B_{j,k} \end{aligned}$$

This method allows us to decompose the conditional expected return of the portfolio  $p$ , conditionally on the sector allocation at date  $t$  in the portfolio.

$$\begin{aligned} E_{t-1}[R_{p,t}] &= \sum_{j=1}^S w_{t-1}^j \alpha_j \\ &\quad + \left( \sum_{j=1}^S w_{t-1}^j B_{j,market} \right) \lambda_{market} \\ &\quad + \left( \sum_{j=1}^S w_{t-1}^j B_{j,size} \right) \lambda_{size} \\ &\quad + \left( \sum_{j=1}^S w_{t-1}^j B_{j,value} \right) \lambda_{value} \\ &\quad + \left( \sum_{j=1}^S w_{t-1}^j B_{j,mom} \right) \lambda_{mom} \end{aligned} \tag{1.13}$$

This expression can be immediately extended to the case of time-varying factor premia. Finally we decompose the conditional portfolio risk at date  $t$ :

$$V_{t-1}[R_{p,t}] = \mathbf{B}'_{t-1} \Sigma_f \mathbf{B}_{t-1} + \sigma_\varepsilon^2$$

where

$$\mathbf{B}'_{t-1} = \begin{bmatrix} \sum_{j=1}^S w_{t-1}^j B_{j,market} \\ \sum_{j=1}^S w_{t-1}^j B_{j,size} \\ \sum_{j=1}^S w_{t-1}^j B_{j,value} \\ \sum_{j=1}^S w_{t-1}^j B_{j,mom} \end{bmatrix}$$

For an easier risk decomposition, we make the assumption that Carhart factors are uncorrelated. This assumption can always be satisfied by applying a suitable rotation to the factors to make them orthogonal (see Section 1.2.2.3). Hence the factors covariance matrix is a diagonal matrix and the risk decomposition is rewritten as follows:

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$$\begin{aligned}
 V_{t-1}[R_{p,t}] = & \left( \sum_{j=1}^s w_{t-1}^j B_{j,market} \right)^2 \times \sigma_{market}^2 \\
 & + \left( \sum_{j=1}^s w_{t-1}^j B_{j,size} \right)^2 \times \sigma_{size}^2 \\
 & + \left( \sum_{j=1}^s w_{t-1}^j B_{j,value} \right)^2 \times \sigma_{value}^2 \\
 & + \left( \sum_{j=1}^s w_{t-1}^j B_{j,mom} \right)^2 \times \sigma_{mom}^2 + \sigma_{\epsilon}^2
 \end{aligned}
 \tag{1.14}$$

Again, introducing time-varying volatilities for factors and specific risk is straightforward.

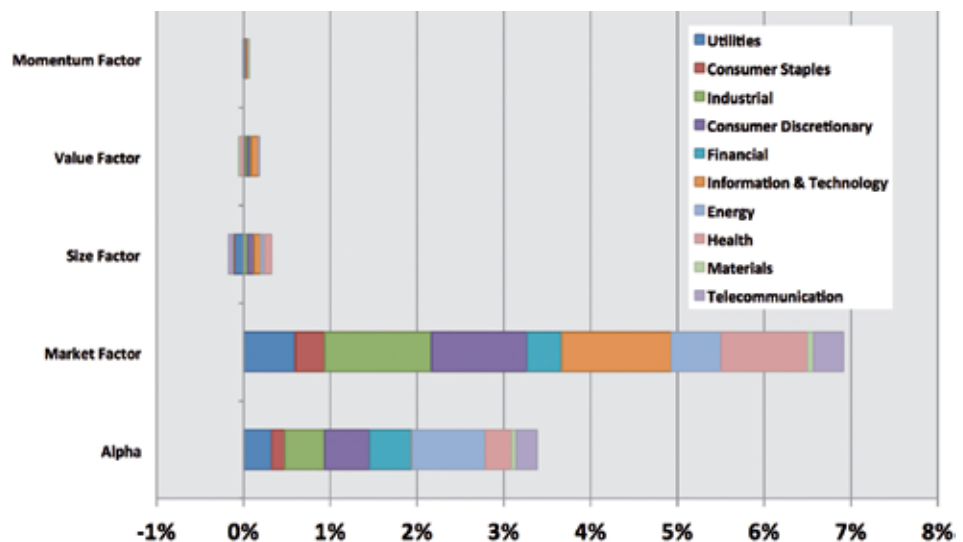
Overall, this method starts by decomposing portfolio performance and risk in factors contributions and then disentangles the factors contributions into sectors contributions. It is a convenient approach for decomposing portfolio performance and risk through two dimensions (factor/sector

or factor/country). But when the number of dimension grows (e.g. by adding country dimension), this method cannot be applied to perform a joint decomposition of return and risk on sectors and countries. Indeed, one has to make a choice in a first stage between projecting the return on sector or on country returns. One solution might be to project the portfolio return on sector-country classes gathering all stocks from a given sector-country pair, but this would severely increase the number of regressors to handle in the first step. In Section 3.1, we will introduce an alternative approach that can accommodate both dimensions simultaneously, by representing alphas and betas as functions of the stock's characteristics.

We illustrate this method by decomposing the expected return and risk of the equally-weighted portfolio of the S&P 500 universe on 20 March 2015. Decompositions are performed for both absolute return and risk. Figure 8 shows the absolute performance

Figure 8: Absolute Performance Decomposition of the Equally-Weighted Portfolio of the S&P 500 Universe on Carhart Factors with Sector Decomposition

Factor returns are from Ken French's library. Sector returns are equally-weighted portfolios from the S&P 500 universe. Returns are quarterly. We regress for each sector portfolios their excess returns on Carhart factor returns for the period 2002–2015 and use OLS to estimate the factors exposures of each portfolios. We then recombine the factors exposures of the equally-weighted portfolio with the portfolio sector allocation at the last rebalancing date (Q1 2015) and the factors exposures of sector portfolios. We use formula (1.13) to make the performance attribution.

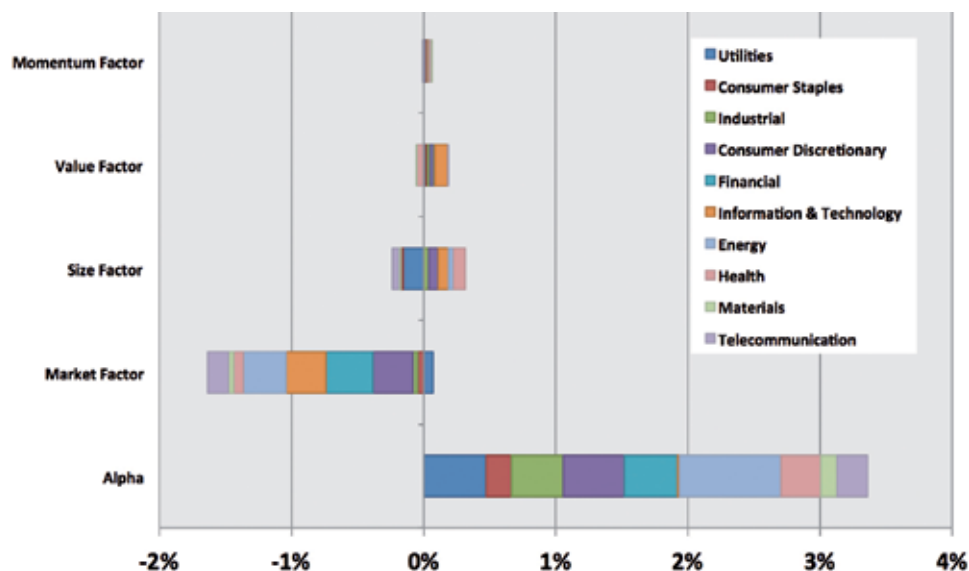


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decomposition. The sum of all factor contributions and the alpha gives the overall performance of the portfolio conditional on the composition of the portfolio at the end of 2015. Most of the performance is explained by market exposure and the alpha. Inside these exposures, some sectors have larger contributions, such as Information and Technology, Industrial, Health for the market exposure and Energy for the alpha. Figure 9 shows similar results in terms of factor contributions and sector decomposition for the excess return of the portfolio over the market factor: the most noticeable difference is the market exposure, which is strongly reduced.

*Figure 9: Relative Performance Decomposition of the Equally-Weighted Portfolio of the S&P 500 Universe on Carhart Factors with Sector Decomposition*

*Factor returns are from Ken French's library. Sector returns are equally-weighted portfolios from the S&P 500 universe. Returns are quarterly. The excess returns to each sector portfolio are regressed on Carhart factors over the period 2002-2015, in order to estimate their exposures. We then obtain the factor exposures of the equally-weighted portfolio in excess of the market factor with the portfolio sector allocation at the last rebalancing date (Q2 2015) and the factor exposures of sector portfolios. We use formula (1.13) to make the performance attribution.*



As appears from Figure 10, most of the realised volatility of the portfolio is due to its exposure to the market and to the sectors that already had the largest contributions to performance (Information and Technology, Industrial and Healthcare). In Figure 11, where we examine the tracking error of the portfolio, it is specific risk that displays the largest contribution.

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Figure 10: Absolute Risk Decomposition Using Euler Decomposition of the Equally-Weighted Portfolio of the S&P 500 Universe on Carhart Factors with Sector Decomposition

Factor returns are from Ken French's library. Sector returns are equally-weighted portfolios from the S&P 500 universe. Returns are quarterly. The excess returns to each sector portfolio are regressed on Carhart factors over the period 2002-2015, in order to estimate their exposures. We then obtain the factor exposures of the equally-weighted portfolio with the portfolio sector allocation at the last rebalancing date (Q2 2015) and the factor exposures of sector portfolios. We use formula (1.14) to make the risk attribution.

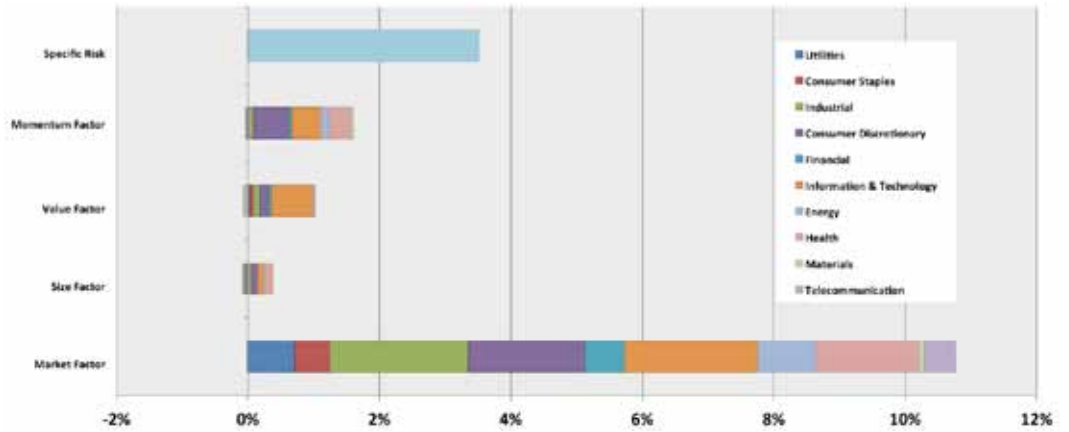
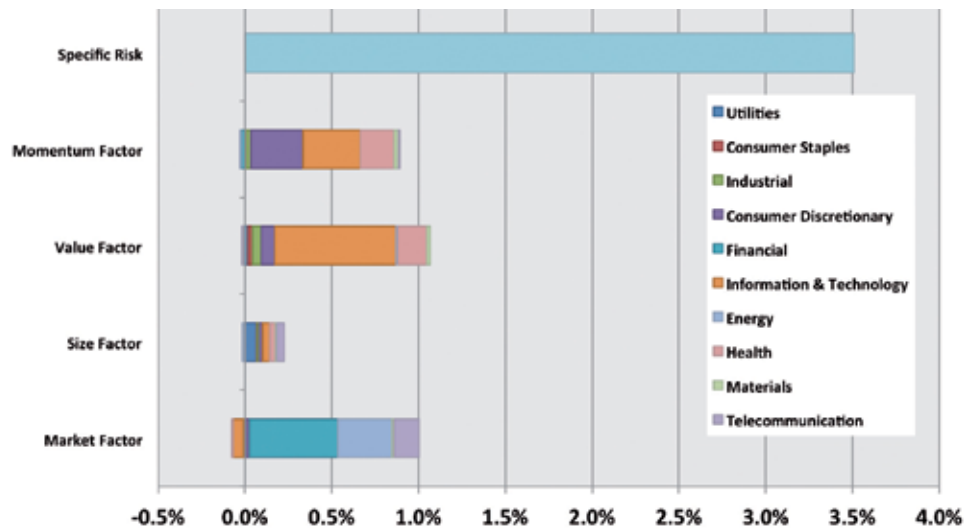


Figure 11: Relative Risk Decomposition Using Euler Decomposition for the Equally-Weighted Portfolio of the S&P 500 Universe on Carhart Factors with Sector Decomposition

Factor returns are from Ken French's library. Sector returns are equally-weighted portfolios from the S&P 500 universe. Returns are quarterly. We regress for each sector portfolios their returns in excess of the market portfolio on Carhart factor returns for the period 2002-2015 and use OLS to estimate the factors exposures of each portfolio. We then recompute the factors exposures of the equally-weighted portfolio in excess of market factor with the portfolio sector allocation at the last rebalancing date (Q2 2015) and the factors exposures of sector portfolios in excess of market portfolio. We use formula (1.14) to make the risk attribution.



## 2. From Historical Betas (and Alphas) to Fundamental Betas (and Alphas)



## 2. From Historical Betas (and Alphas) to Fundamental Betas (and Alphas)

Asset managers need to estimate the sensitivity of each stock to the market in order to implement an allocation consistent with their views about factor returns or lack thereof. Specifically, they must estimate the beta of each stock conditional on the information available to date. Mathematically, this quantity is defined as:

$$\beta_{i,t-1} = \frac{\text{Cov}[R_{i,t}, R_{m,t} | \Phi_{t-1}]}{\text{Var}[R_{m,t} | \Phi_{t-1}]}, \quad (2.1)$$

where  $R_{i,t}$  denotes the return on stock  $i$  in period  $[t-1, t]$  in excess of risk-free rate,  $R_{m,t}$  is the excess return on the market portfolio and  $\Phi_{t-1}$  is the information set available at date  $t-1$ . The traditional measure of conditional market exposure is the beta estimated over a sample period, but if the distributions of stock and market returns change over time, the sample estimates are not good estimators of the true conditional moments. By shifting the sample period (rolling-window estimation), one generates time dependency in the beta, but due to the overlap between estimation windows, the historical beta changes relatively slowly.

This section is precisely devoted to the construction of an alternative estimator for the conditional beta. We name this estimator a "fundamental beta" because it is defined as a function of the stock's fundamental characteristics. In Section 2.1, we start by reviewing the competing approaches to estimate market betas that have been proposed in the literature. Next, we define and estimate two versions of the fundamental beta in Section 2.2 and Section 2.3.

### 2.1 Measuring Market Beta

The traditional approach to measuring the market exposure of a stock or a portfolio is

to run a time-series regression of the stock (excess) returns on a market factor (excess return to a proxy for the market portfolio). The exact choice of the factor depends on the application of the measure. In equity portfolio management, a fund manager is concerned with the exposure to the stock market, and a broad cap-weighted stock index is a good representation of market movements. For asset pricing purposes, more care is needed in the definition of the market factor. Indeed, in the CAPM of Sharpe (1964), the market portfolio represents the aggregate wealth of agents, which includes not only stock holdings but also non-tradable assets such as human capital. This point is related to Roll's (1977) critique of empirical tests of the CAPM, which are joint tests of the model itself and of the quality of the proxy used for the unobservable true market portfolio.

Once a suitable proxy has been specified, one has to estimate the conditional beta. This is done in general by running a regression of stock returns on the market over a rolling window. If the joint distribution of stock and market returns were constant over time, the sample beta at date  $t-1$  would be a consistent estimator of the conditional beta on this date, and the variation in rolling-window estimates would be due to sampling errors only. But returns are not identically distributed: there are clusters in volatility (Harvey, 1989; Bollerslev, Engle and Nelson, 1994) and stock returns exhibit some predictability (Fama, 1981; Keim and Stambaugh, 1986; Fama and French, 1989; Cochrane, 2008), which is another way of saying that expected returns are not constant. Moreover, the beta itself is not constant (Rosenberg and Marathe, 1976). This has led researchers to look for alternative techniques to measure the

## 2. From Historical Betas (and Alphas) to Fundamental Betas (and Alphas)

beta otherwise than by regressing stock or portfolio returns on a market proxy.

### 2.1.1 Alternatives to Regression Technique

Wang and Menchero (2014) argue that the market exposure of a stock is an aggregation of the exposures to multiple factors including country, sector and investment styles. Formally, stock returns are regressed on the  $K$  risk factors:

$$R_{i,t} = \alpha_i + \sum_{k=1}^K b_{k,i} F_{k,t} + \varepsilon_{i,t},$$

so the beta can be decomposed as:

$$\beta_i = \frac{\text{Cov}[R_{m,t}, R_{i,t}]}{\sigma_m^2} = \sum_{k=1}^K b_{k,i} \frac{\text{Cov}[F_{k,t}, R_{m,t}]}{\sigma_m^2} + \frac{\text{Cov}[\varepsilon_{i,t}, R_{m,t}]}{\sigma_m^2}.$$

This can be rewritten as:

$$\beta_i = \sum_{k=1}^K b_{k,i} \times \frac{\sigma_{Fk}^2 \beta_{Fk}}{\sigma_m^2} + \frac{\sigma_{\varepsilon i}^2 \beta_{\varepsilon i}}{\sigma_m^2},$$

where  $\beta_{Fk}$  is the beta of the market factor with respect to factor  $k$  and  $\beta_{\varepsilon i}$  is the beta of the specific stock return. This beta is called by Wang and Menchero a "predicted beta", although there is no explicit modelling for the time variation. It is intended as a better estimate for the conditional market exposure in that it takes into account the fundamental sources of risk that determine the market exposure. In the empirical application, the multi-factor model is a version of the Barra model (see Section 1.3.1), where the betas  $b_{k,i}$  are fixed according to stock's characteristics such as nationality, type of activity, size, liquidity, volatility, dividend yield, book-to-price ratio, etc. Hence, the predicted beta is a function of some fundamental characteristics of the stock.

Other authors have sought to capture the determinants of time variation in the betas. A first idea to generate a time-varying beta is to introduce a dynamic model. Ghysels (1998) examines various parametric models, including those of Ferson (1990), Ferson and Harvey (1991, 1993) and Ferson and Korajczyk (1995), but showed that these models are less accurate and estimate highly volatile betas. Indeed, they tend to produce large pricing errors and may have tendency to overstate the beta time variation.

### 2.1.2 Using Instrumental Variables

Ferson and Schadt (1996) take a different approach by summarising conditioning information in the current value of a state vector  $\mathbf{Z}$  and assuming a linear specification for the beta:

$$\beta_{it} = \beta_i(\mathbf{Z}_t) = \mathbf{b}_i^0 + \mathbf{B}_i' \mathbf{z}_t,$$

where  $\mathbf{z}_t = \mathbf{Z}_t - E(\mathbf{Z})$  is a vector of the deviations of  $\mathbf{Z}_t$  from the unconditional means.  $\mathbf{b}_i^0$  is the unconditional mean of the conditional beta and can be interpreted as an "average beta":  $\mathbf{b}_i^0 = E[\beta_i(\mathbf{Z}_t)]$ . The elements of  $\mathbf{B}_i$  are the response coefficients of the conditional beta to the information variables  $\mathbf{Z}_t$ . In general, the average conditional beta does not equal the unconditional beta, obtained by replacing the conditional moments in (2.1) by their unconditional counterparts. We show in Appendix A1 that a sufficient condition for this equality to hold is that the conditional expected return and the conditional variance of the market be constant.

Ferson and Schadt use the time-varying beta to construct a conditional CAPM, that is a CAPM in which proportionality between expected returns and the market beta holds period-by-period in terms of



## 2. From Historical Betas (and Alphas) to Fundamental Betas (and Alphas)

conditional moments, as opposed to being true unconditionally:

$$E[R_{i,t}|\Phi_{t-1}] = \beta_{i,t-1} \times E[R_{m,t}|\Phi_{t-1}]. \quad (2.2)$$

The authors find evidence that alpha is closer to zero in a conditional model than in an unconditional model, which suggests that the conditional CAPM does a better job of explaining average abnormal returns. For the predetermined state vector, the authors use economic variables with a one-month lag: the Treasury bill yield, the dividend yield of the CRSP value-weighted New York Stock Exchange (NYSE) and American Stock Exchange (AMEX) stock index, the slope of the term structure, the quality spread in the corporate bond market, and a dummy variable for the month of January. By using a sample of 67 open-end mutual funds over 23 years, they show that traditional measures of average performance (Jensen's alpha) are more often negative than positive for mutual funds, which has been interpreted as inferior performance. Both a simple CAPM and a four-factor model produce this result in their sample, but with the conditional model, the distribution of alphas shifts to the right and is centred around zero. Shanken (1990) uses one-month bill rate and its volatility as characteristics for the public information vector. The author estimates parameters with OLS regression and tests if any of the beta decomposition coefficients is non-zero. He insists on using corrected standard errors because of the presence of conditional heteroskedasticity and use White estimator<sup>7</sup> (corrected standard errors are larger than usual OLS standard errors).

7 - White estimator provides heteroskedasticity consistent standard errors that are robust to general forms of temporal dependence but not to spatial dependence, unlike the estimator of Driscoll and Kraay (1998).

The two main steps for building conditional models are the identification of the information vector  $\mathbf{Z}_t$  and the estimation of the response coefficients of the conditional beta to the information variables  $\mathbf{Z}_t$ . In this process, one has to take care of the risks of data mining and spurious regression, as Ferson, Sarkissian and Simin (2006) demonstrate. Data mining refers to the practice of searching through the data to find predictor variables, and spurious regression arises with the persistence of high autocorrelation of a predictor variable, which creates artificially significant relationships.

### 2.1.3 Using Microeconomic Variables

In this paper, we follow the linear model for the beta introduced by Ferson and Schadt (1996), but we replace the macroeconomic content by microeconomic variables, that are variables specific to each stock. Specifically, we choose, for the information vector  $\mathbf{Z}_t$ , the following firm's attributes:  $Cap_{i,t}$  (market capitalisation of firm  $i$ ),  $Bmk_{i,t}$  (the book-to-market ratio of firm  $i$ ) and  $Ret_{i,t}$  (past 1-year return of firm  $i$ ) in direct line with the construction of Fama and French (1993) factors augmented by the momentum factor (Carhart, 1997). This makes the conditional beta a function of several stock's characteristics, as in Wang and Menchero (2014). A difference between their approach and ours is that we have only three characteristics while the Barra-type model on which they rely requires a substantial amount of additional information such as geographical and industry classification, liquidity, volatility and dividend yield. Moreover, we do not rely on a multi-factor model to estimate the market beta.

## 2. From Historical Betas (and Alphas) to Fundamental Betas (and Alphas)

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There are several justifications for our choice of conditioning information. First, several studies have shown that market risk is linked to accounting variables. Beaver, Kettler and Scholes (1970) use the following accounting variables: dividend payout (dividend/earning), growth, leverage, liquidity, asset size, variability of earnings, and covariability of earnings within the context of 'intrinsic value' analysis. They find high degree of contemporaneous association between accounting risk measure and the market risk exposure. Jarvela, Kozyra and Potter review and update this study in 2009. With OLS estimation conducted on 222 randomly selected publicly traded companies, they determine the correlation between the risk measures related to accounting variables (earnings variability, dividend payout, and leverage) and those determined by the market. Their results confirm those of Beaver, Kettler and Scholes in 1970 and show correlation between market risk and accounting risk measures. They conclude that accounting information is a possible alternative to market risk information.

Second, our choice of state variables echoes of course the definition of the Fama-French and Carhart factors. These factors are empirical pricing factors that account for several of the empirical regularities that are left unexplained by the CAPM: rather unsurprisingly, they capture the size and the value effects and the continuation of short-term returns, but together with the market factor, the size and the value factors are also able to capture the return spread across portfolios formed on earnings-to-price, cash flow-to-price, past sales growth and past long-term returns (Fama and French, 1996). In spite of this empirical success, there is no undisputed economic explanation for why exposure to these factors should be rewarded with

higher expected returns, and some papers question their role in asset pricing.

Ang and Chen (2007) show that in a conditional version of the CAPM, the long-term value premium is no longer statistically significant. In a related contribution, Bandi et al. (2010) study long-run relation between expected returns and market beta risk in the post-1963 period. Using Fama and MacBeth (1973) cross-sectional-regression they estimate the market risk premium over a five to ten year period at economically-reasonable values between 6 and 11% per annum. Conversely, size and value factors become less statistically significant. Market risk factor increases its statistical and economic relevance with the horizon and appears in the long run to be the major contributing factor. Value and size, which are still broadly considered as important risk factors when adopting a short-term perspective, become less statistically significant with long horizon.

Overall, there is currently no definitive evidence that the size and value factors are economically justified asset pricing factors and that they are required to explain the cross-section of expected returns. Thus, as a first approximation, it is not unreasonable to stay in the context of a one-factor model in order to design a conditional asset pricing model. Since there are size, value and momentum premia that the standard CAPM cannot explain, we include size, value and momentum scores in the set of instrumental variables in an attempt to capture these premia through the market beta.

In what follows, we measure a "fundamental alpha" and a "fundamental beta" expressed as functions of the three aforementioned scores. The objective of this construction is to achieve a better estimation of the

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conditional alpha and beta than what would be obtained with rolling-window regressions: indeed, changes in firm's characteristics are only slowly reflected in rolling-window estimates, while they are instantaneously incorporated in the fundamental parameters. In our tests of the one-factor model, we will also ask whether a conditional CAPM based on the fundamental beta can explain the size, value and momentum effects.

### 2.2 First Specification of Fundamental Beta

An important difference between the conditional beta of Ferson and Schadt (1996) and ours is that their state vector  $\mathbf{Z}_t$  consists of variables that are identical for all stocks while we include microeconomic data. Hence, in Ferson and Schadt's model, cross-sectional variation in the beta comes only from the differences in the coefficients  $\mathbf{b}_i^0$  and  $\mathbf{B}_i$  across stocks, while our model, such variation can be generated even with uniform coefficients. We thus study separately two versions of the conditional beta. The first one, which we present in this subsection, has the same coefficients for all stocks, while the second one, which we develop in Section 2.3, relaxes this restriction.

#### 2.2.1 Model Formulation

We consider the following one-factor model for stock returns, in which the alpha and the beta are functions of the three observable attributes that define the Fama-French-Carhart factors: market capitalisation ( $Cap_{i,t}$ ), book-to-market ratio ( $Bmk_{i,t}$ ) and past 1-year return ( $Ret_{i,t}$ ) for the stock  $i$  at date  $t$ . Hence we have the following relations:

$$R_{i,t} = \alpha_{i,t} + \beta_{i,t} (Rm_t) + \varepsilon_{i,t}$$

$$\alpha_{i,t} = \theta_{\alpha,0} + \theta_{\alpha,Cap} \times Cap_{i,t} + \theta_{\alpha,Bmk} \times Bmk_{i,t} + \theta_{\alpha,Ret} \times Ret_{i,t}$$

$$\beta_{i,t} = \theta_{\beta,0} + \theta_{\beta,Cap} \times Cap_{i,t} + \theta_{\beta,Bmk} \times Bmk_{i,t} + \theta_{\beta,Ret} \times Ret_{i,t}$$

The conditional beta  $\beta_{i,t}$  of a stock  $i$  at period  $t$  is a measure of the market exposure over the next period conditional on the attributes at date  $t$ . In the above model, stocks that have the same attributes on a given date will also have the same market exposure at this instant. But their market exposures can diverge at subsequent dates as their attributes become different. We call  $\beta_{i,t}$  a "fundamental beta" because it is a function of variables measured at the stock level. As is clear from the model equations, there are eight coefficients to estimate. The coefficients  $\theta_{\alpha,0}$  and  $\theta_{\beta,0}$  are respectively the "alpha intercept" and the "beta intercept" and the other parameters represent the sensitivities of the alpha and the beta with respect to the characteristics.

The mean of the conditional beta (or "average beta") is:

$$\bar{\beta}_i = \theta_{\beta,0} + \theta_{\beta,Cap} \sum_{t=1}^T \frac{1}{T} Cap_{i,t} + \theta_{\beta,Bmk} \sum_{t=1}^T \frac{1}{T} Bmk_{i,t} + \theta_{\beta,Ret} \sum_{t=1}^T \frac{1}{T} Ret_{i,t}$$

This "average beta" is different from the unconditional beta (or "historical beta") estimated with OLS regression over the entire sample.

To obtain the conditional beta of a portfolio, it suffices to use the fact that

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the beta is linear in the vector of returns: the fundamental beta of the portfolio is the weighted sum of the fundamental betas of the constituents:

$$\begin{aligned} \beta_{p,t} = \sum_i w_{i,t} \beta_{i,t} = & \theta_{\beta,0} + \theta_{\beta, \text{Cap}} \underbrace{\sum_i w_{i,t} \text{Cap}_{i,t}}_{=\text{Cap}_{p,t}} \\ & + \theta_{\beta, \text{Bmk}} \underbrace{\sum_i w_{i,t} \text{Bmk}_{i,t}}_{=\text{Bmk}_{p,t}} \\ & + \theta_{\beta, \text{Ret}} \underbrace{\sum_i w_{i,t} \text{Ret}_{i,t}}_{=\text{Ret}_{p,t}} \end{aligned}$$

where  $(\text{Cap}_{p,t})$ ,  $(\text{Bmk}_{p,t})$  and  $(\text{Ret}_{p,t})$  define the size, value and momentum scores of the portfolio at period  $t$ . This method is "holding-based": it requires knowledge of portfolio composition and weights, in addition to the constituents' attributes and the model coefficients.

### 2.2.2 Estimation

Our estimation procedure is the Generalised Calendar Time (GCT) method introduced by Hoechle, Schmidt and Zimmermann (2015). In the traditional Calendar Time approach, stocks are first sorted in portfolios on some attribute and each portfolio is then regressed against a set of factors in order to study the relationship between the abnormal performance and the attribute. In contrast, the GCT model represents the alpha and the beta of each stock as a function of one or more attribute(s), which can be discrete or continuous variables. Thus, a continuous attribute, such as those that we use in the computation of the fundamental beta, needs not be discretised. This avoids the loss of information incurred by ignoring differences across stocks that belong to the same group. As shown by Hoechle, Schmidt and Zimmermann (2015), the GCT approach nests the Calendar

Time approach in the sense that it can reproduce the exact results obtained by sorting stocks in portfolios.

In the GCT method, regression of returns is performed at the stock level, and the exogenous variables are the products of the market factor with all the characteristics included in the model.<sup>8</sup> These interaction terms arise because the factor exposure (the beta) is represented as a linear function of the characteristics. In vector form, the decomposition of stock returns in the one-factor model reads:

$$R_{i,t} = [\mathbf{Z}_{i,t} \mathbf{R}_{m,t}] \boldsymbol{\theta} + \varepsilon_{i,t} \quad (2.4)$$

Here,  $R_{i,t}$  denotes the return at period  $t$  of an individual stock  $i$ ,  $\mathbf{Z}_{i,t}$  is the vector of stock characteristics (1×3-dimensional vector), and  $\varepsilon_{i,t}$  denotes the idiosyncratic return, assumed to be centred and uncorrelated from the exogenous variables. Overall, regression (2.4) comprises a total of six explanatory variables and eight coefficients which are stored in the vector  $\boldsymbol{\theta}$ . While the subject characteristics in vector  $\mathbf{Z}_{i,t}$  may vary across both the time dimension and the cross-section, the market factor varies over time but not across stocks. The estimation of the eight coefficients is done by pooling times series and cross-sectional information in a panel regression: the sum of squared residuals  $\varepsilon_{i,t}^2$  over all stocks and dates is minimised with respect to the eight parameters. We then compute Driscoll and Kraay (1998) nonparametric covariance matrix estimator, which produces heteroskedasticity and auto correlation consistent standard errors that are robust to general forms of spatial and temporal dependence. Prior to the regression, each attribute is transformed into a z-score according to the following formula:

$$z\text{-score}\{\text{attribute}\}_{i,t} = \frac{\text{attribute}_{i,t} - \text{mean}_t[\text{attribute}]}{\text{std}_t[\text{attribute}]}$$

<sup>8</sup> - The GCT method can also be employed with several risk factors, like in the Fama-French model.

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where  $z\text{-score}\{attribute\}_{i,t}$  denotes the normalised score of asset  $i$  for the given attribute,  $attribute_{i,t}$  denotes the attribute value of asset  $i$ ,  $mean_t [attribute]$  denotes the mean value of the attribute across the whole universe of stocks at fixed period and  $std_t [attribute]$  denotes the standard deviation of the attribute across the whole universe of stocks at fixed period. The result is that each attribute has a null mean and a standard deviation equal to 1 at each period. The pooled model is estimated over the S&P 500 universe with 500 stocks and the period 2002-2015 (51 quarterly returns), hence  $51 \times 500 = 25,500$  observations.

Because z-scores are centred, the fundamental alpha and beta of an equally-weighted portfolio of all stocks are constant and equal to  $\theta_{\alpha,0}$  and  $\theta_{\beta,0}$ .

$$\begin{aligned} \beta_{p,t} = & \theta_{\beta,0} + \theta_{\beta,cap} \sum_i \frac{1}{N} Cap_{i,t} \\ & + \theta_{\beta,bmk} \sum_i \frac{1}{N} Bmk_{i,t} \\ & + \theta_{\beta,ret} \sum_i \frac{1}{N} Ret_{i,t} = \theta_{\beta,0} \end{aligned}$$

In Table 1, we verify that these estimates are close to the in-sample alpha and beta of the broad EW portfolio. Table 2 displays the complete set of parameter estimates, together with the standard errors and p-values, estimated via the standard OLS formulas or via the Driscoll-Kraay covariance matrix. We observe large differences between traditional t-Stats

and the corrected t-Stats obtained with the Driscoll and Kraay methodology. Except for the coefficient  $\theta_{\alpha,Ret}$ , all the coefficients are significant. But t-Stats are reduced with the Driscoll and Kraay methodology and some coefficients, in particular alpha components as  $\theta_{\alpha,Bmk}$ , appear non-significant. But the beta decomposition is still meaningful because the beta components are still significant with the corrected t-Stats.

### 2.2.3 Beta as a Function of Attributes

Figures 12 and 13 present the results of Table 2 on fundamental beta, first as a function of the two microeconomic characteristics on which the Fama-French size and value factors are based and then as a function of the microeconomic characteristic on which the momentum factor is based. Under the assumption of a standard normal distribution for the z-score variable, almost all the observations (99.73%) lie between -3 and 3, so we restrict the range in each axis to [-3;3]. In this range, fundamental beta lies between 0.4 and 1.4 which is a right scale when considering market exposures. We also see that the fundamental beta is decreasing in market cap and past 1-year return and increasing in book-to-market ratio.

These observations can be related to the signs of the size, value and momentum premia in the period under study. Suppose that the conditional CAPM with the fundamental beta holds, that is the conditional expected return on a stock is

Table 1: Comparison of Fundamental and Historical Betas for the S&P 500 Equally-Weighted Portfolio  
The in-sample alpha and beta of the S&P 500 equally-weighted portfolio are estimated by regressing quarterly index returns on market returns from Ken French's library over the period 2002-2015. The coefficients  $\theta_{\alpha,0}$  and  $\theta_{\beta,0}$  are obtained by a pooled regression of the 500 stock returns.

Abnormal Return		Market Exposure	
Coefficient $\theta_{\alpha,0}$	In-sample $\alpha$	Coefficient $\theta_{\beta,0}$	In-sample $\beta$
0.051	0.05	0.919	0.92

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Table 2: Fundamental Alpha and Beta Coefficients Estimated with the GCT-Regression Model on the S&P 500 Universe over the period 2002-2015

The coefficients of the one-factor model are estimated with a GCT-regression of the 500 stocks from the S&P 500 universe. Data is quarterly and spans the period 2002-2015, and market returns are from Ken French's library. Attributes (capitalisation, book-to-market and past one-year return) come from the ERI Scientific Beta US database and are updated quarterly.

	Estimate	t-Stat	p-Value	Driscoll-Kraay t-Stats	Corrected p-Value
$\theta_{\alpha,0}$	0.051	12.81	0	1.79	0.073
$\theta_{\alpha,Cap}$	0.011	1.89	0.059	0.73	0.466
$\theta_{\alpha,Bmk}$	-0.014	-4.39	0	-2.72	0.007
$\theta_{\alpha,Ret}$	0.008	1.32	0.187	0.85	0.398
$\theta_{\beta,0}$	0.919	66.48	0	7.10	0
$\theta_{\beta,Cap}$	-0.136	-7.00	0	-2.52	0.012
$\theta_{\beta,Bmk}$	-0.053	-4.80	0	-2.79	0.005
$\theta_{\beta,Ret}$	0.060	2.44	0.015	2.03	0.042

proportional to its conditional beta:

$$E_t[R_{i,t+1}] = \beta_i(\mathbf{Z}_t) \times E_t[Rm_{t+1}],$$

where  $E_t$  is the expectation conditional on information available at date  $t$ . This equation constitutes the "fundamental CAPM", which we study in more detail in Section 3.3. Because the realised market premium was positive (8.6% per year) in the period 2002-2015, the fundamental CAPM predicts that small stocks have higher expected returns than large stocks, value stocks should outperform growth stocks, but past winners should underperform past losers. The first two predictions are in line with the evidence in favour of the size and the value effects, and they are verified in our dataset since the ex-post size and value premia were respectively 2.9% and 0.8%.

On the other hand, the last prediction seems to be at odds with the momentum effect, because it is past winners that outperform past losers (Jegadeesh and Titman, 1993). However, in this particular sample, it happens that past winners did underperform past losers by 0.4% per year. Hence, the prediction of a "reversed" momentum effect implied by Figure 13 is

consistent with the in-sample evidence. The conclusion is that the fundamental CAPM can explain, at least partially, the size, the momentum and the value effects in returns only with the market exposure. We emphasise that at this stage, we have only verified that the model predicts size, value and momentum premia that have the right signs, but we have not shown that it predicts their magnitude. In this sense, the explanation of the three effects may only be partial. In Section 3.3, we conduct a series of asset pricing tests in order to test the ability of the model to account for these effects in full, that is to explain the cross-section of average returns with the cross-section of market returns. We will focus later on this point and show that the conditional CAPM does a better job than the "static" CAPM and even the Carhart four-factor model to explain the cross-section of average returns.

Hence our approach rather to consider attribute as an effective way to construct factor mimicking portfolios, allows to sort stocks with similar properties. Stocks with the same attributes' level at a fixed period will have the same market exposure for this period. But their market exposures



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Figure 12: Fundamental Beta as a Function of Firm's Attributes (Book-to-Market ratio and Market Capitalisation)  
 Book-to-market ratio and market capitalisation coefficients are estimated with the GCT-regression model on the 500 stocks from the S&P 500 universe with quarterly stock returns, quarterly z-scores attributes and quarterly market returns from Ken French's library over the period 2002-2015. Attributes come from the ERI Scientific Beta US database and are updated quarterly.

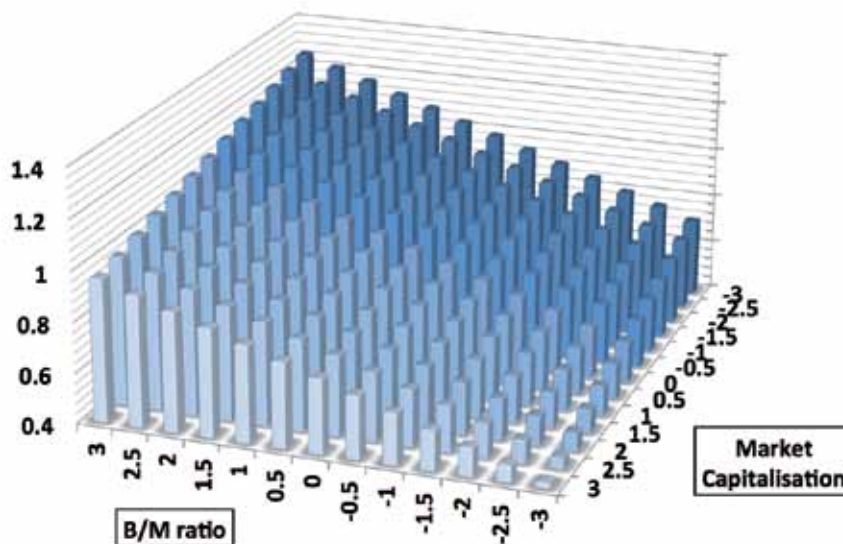
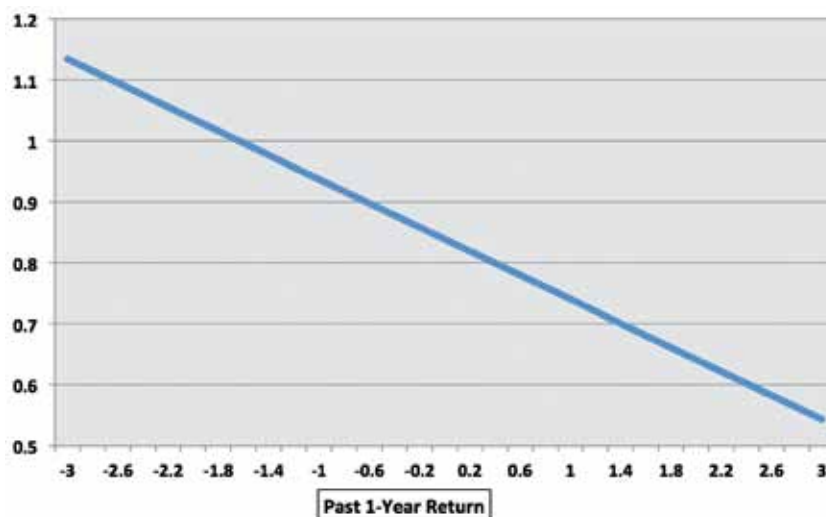


Figure 13: Fundamental Beta as a Function of Stock's Past 1-Year Return  
 Past 1-year return coefficient is estimated with the GCT-regression model on the 500 stocks from the S&P 500 universe with quarterly stock returns, quarterly z-scores attributes and quarterly market returns from Ken French's library over the period 2002-2015. Attributes come from the ERI Scientific Beta US database and are updated quarterly.



can diverge through the time as their attributes can change in different ways.

### 2.2.4 Stability of Model Parameters

The parameters  $\theta$  of the one-factor model are assumed to be constant over time. This assumption may be questioned, so it is important to assess the stability

of coefficients over time. To this end, we repeat the panel regression over a rolling window of a fixed size through the sample. If parameters remain constant over the entire sample, then the variation in estimates across the different windows should only reflect statistical noise. But if the parameters change at some point



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during the sample, then the rolling-window estimates should capture this instability. We check the stability of the coefficients from the GCT-regression model by using 5-year rolling windows of quarterly data.

From Figure 14, the coefficients appear to be stable during the whole period except during the sub-period going from end of 2007 to 2009, in which the book-to-market ratio and the one-year price return coefficients change sign and the intercept coefficient increases from 0.8 to 1.3. This instability can be related to the rise in uncertainty caused by the subprime mortgage crisis of 2007. Except for this sub-period and the year 2014, the coefficients remain approximately constant over the entire period. At least, no trend is visible that would suggest that a permanent change in the model occurred in this sample. Overall, the model coefficients are relatively stable over time. In the next section, we show however that the assumption of uniform coefficients across stocks is questionable.

### 2.3 A More Flexible Specification for the Fundamental Beta

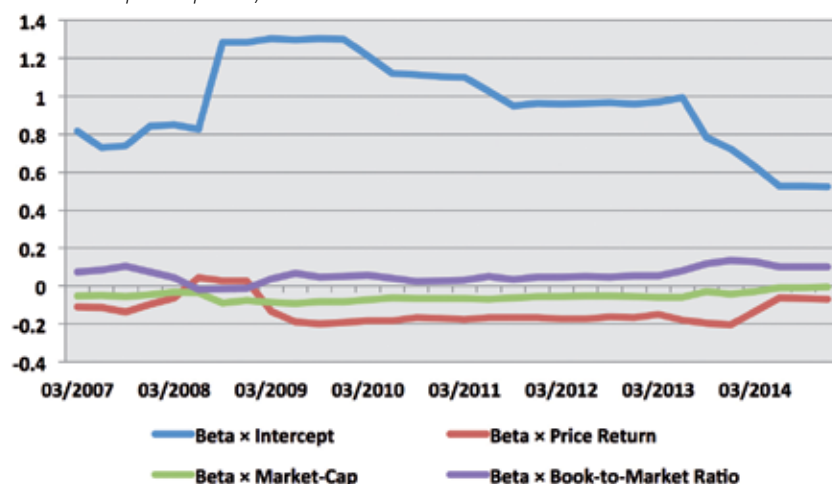
The one-factor model introduced in Section 2.2 has exactly eight parameters: four of them tie the alpha to the characteristics, and the other four correspond to the beta. As a result, the sensitivities of the fundamental alpha and beta with respect to the characteristics are identical for all stocks. This can be regarded as a strong restriction, so we now present a more flexible version of the model in which these effects can be different from one stock to the other. The model is written as follows:

$$R_{i,t} = \alpha_{i,t} + \beta_{i,t} \times R_{m,t} + \varepsilon_{i,t}$$

$$\alpha_{i,t} = \theta_{\alpha,0,i} + \theta_{\alpha,Cap,i} \times Cap_{i,t} + \theta_{\alpha,Bmk,i} \times Bmk_{i,t} + \theta_{\alpha,Ret,i} \times Ret_{i,t}$$

$$\beta_{i,t} = \theta_{\beta,0,i} + \theta_{\beta,Cap,i} \times Cap_{i,t} + \theta_{\beta,Bmk,i} \times Bmk_{i,t} + \theta_{\beta,Ret,i} \times Ret_{i,t}$$

Figure 14: One-Factor Model Coefficients Estimated With a Panel Regression on 5-Year Rolling Windows With Quarterly Step  
The coefficients are estimated with the GCT-regression model on the 500 stocks from the S&P 500 universe with quarterly stock returns, quarterly z-scores attributes and quarterly market returns from Ken French's library over the period 2002-2015. We use 5-year rolling windows of quarterly data to obtain time-varying coefficients each quarter. Attributes come from the ERI Scientific Beta US database and are updated quarterly.



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These quantities are still called a "fundamental alpha" and a "fundamental beta". For  $N$  stocks, the model has  $8N$  coefficients to estimate instead of 8 with the previous model. The increase in the number of parameters has two effects. On the one hand, misspecification risk is reduced because restrictions on parameters are relaxed; on the other hand, we lose degrees of freedom, which may cause loss of robustness.

As in the constrained case, the model is estimated by minimising the sum of squared residuals  $\epsilon_{i,t}$  over all dates and stocks. But because the coefficients are independent from one stock to the other, the pooled regression is equivalent to  $N$  time-series regressions:

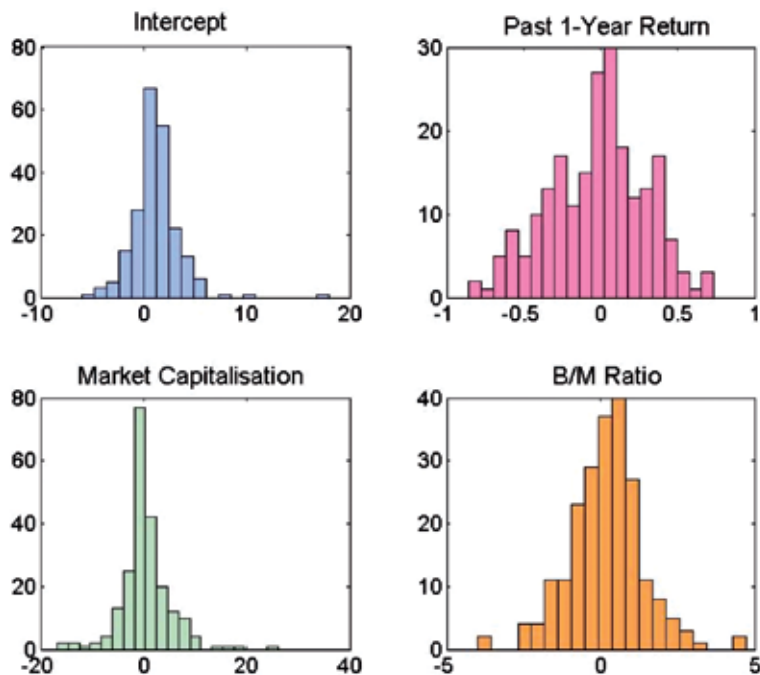
$$\begin{aligned} &\text{minimising } \sum_{i,t} \epsilon_{i,t}^2 \text{ is equivalent to} \\ &\text{minimising } \sum_t \epsilon_{i,t}^2 \text{ for each } i \end{aligned}$$

Hence, the coefficients can be estimated separately for each stock, by running a time-series regression. We regress for each stock its excess return on the market return and the market return crossed with the stock's attributes. For a stock  $i$ , the regression equation takes the form:

$$\begin{aligned} R_{i,t} = &\theta_{\alpha,0,i} + \theta_{\alpha, \text{cap},i} \times (\text{Cap}_{i,t}) + \theta_{\alpha, \text{Bmk},i} \times (\text{Bmk}_{i,t}) \\ &+ \theta_{\alpha, \text{Ret},i} \times (\text{Ret}_{i,t}) + \theta_{\beta,0,i} \times R_{m,t} \\ &+ [\theta_{\beta, \text{cap},i} \times (\text{Cap}_{i,t}) + \theta_{\beta, \text{Bmk},i} \times (\text{Bmk}_{i,t}) \\ &+ \theta_{\beta, \text{Ret},i} \times (\text{Ret}_{i,t})] \times R_{m,t} + \epsilon_{i,t} \end{aligned} \tag{2.5}$$

The model is estimated over the S&P 500 universe with  $N = 500$  stocks and the period 2002-2015 (which corresponds to 51 quarterly returns). Hence we obtain 500 coefficients of each type (intercept, capitalisation sensitivity, book-to-market sensitivity and past return sensitivity) for the alpha, and 500 others for the beta.

Figure 15: Distributions of Coefficients in the More Flexible One-Factor Model  
The coefficients are estimated through time-series regressions for each of the 500 stocks from the S&P 500 universe with quarterly stock returns, z-score attributes and market returns from Ken French's library over the period 2002-2015. Attributes come from the ERI Scientific Beta US database and are updated quarterly.



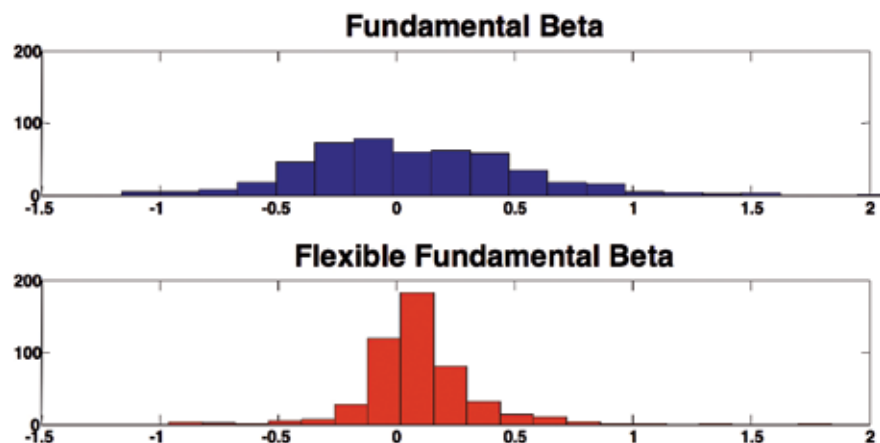
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Figure 15 displays the distributions of the four coefficients that appear in the decomposition of the fundamental beta. For each coefficient, there is a substantial dispersion in the estimates across the 500 stocks, which suggests that the model with uniform coefficients imposes indeed too much structure.

Even if the fundamental beta diverges from in-sample historical beta during sub-period, we expect in average over the period that the fundamental beta would be close to the historical beta. For this reason, we study the difference of the average fundamental beta over the period with the historical in-sample beta for each of the 500 stocks. Results are reported in Figure 16 for the more flexible model and for the constrained version.

Specific coefficients from model of formula 2.4 lead to a fundamental beta closer to the in-sample historical beta. Indeed, Figure 16 shows that the "Flexible Fundamental Beta" is less dispersed around the in-sample historical beta.

*Figure 16: Temporal Mean of Fundamental Beta and "Flexible" Fundamental Beta Minus Historical Beta*  
The coefficients are estimated with the GCT-regression model for the Fundamental Beta and with time-series regressions for the Flexible Fundamental Beta on the 500 stocks from the S&P 500 universe with quarterly stock returns. We use z-score attributes from the ERI Scientific Beta US universe and market returns come from Ken French's library over the period 2002-2015. We represent for the 500 stocks, the difference between the average over the period of the fundamental and Flexible Fundamental Beta and the in-sample historical beta. We obtain two distributions of 500 stocks for each estimation method.



## 2. From Historical Betas (and Alphas) to Fundamental Betas (and Alphas)

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# 3. Applications of Fundamental Beta



## 3. Applications of Fundamental Beta

In this section, we present three applications of the fundamental beta. We first use this approach to embed the sector dimension in our multi-factor risk and performance analysis. In complement to the previous methods presented in Section 1.3.2 and Section 1.3.3, we show that the fundamental beta approach is more convenient when the multi-factor analysis is extended to additional dimensions (e.g. sector and regions).

We then compare the fundamental and the rolling-window betas as estimators of the conditional beta, by constructing market-neutral portfolios based on the two methods. We show that the fundamental method results in more accurate estimates of market exposures, since the portfolios constructed in this way achieve better ex-post market neutrality than those in which the beta was estimated by regressing past stock returns on the market.

The third application is a comprehensive asset pricing test to compare the conditional CAPM with two standard alternative factor models: the unconditional CAPM, where the beta and the market premium are constant, and the multi-factor Carhart model. We follow the Fama and MacBeth (1973) cross-sectional method and we compare the alphas of the equally-weighted portfolios sorted on book-to-market ratio, market capitalisation, and past one-year return under the three models. We show that the introduction of fundamental betas is usefully complemented by that of a time-varying market premium, which further reduces the alphas. The conditional model based on fundamental betas proves to be the most effective at explaining the cross-section of expected returns.

### 3.1 Introducing Sectors in Multi-Dimensional Risk and Performance Analysis

Risk and performance analysis for equity portfolios is most often performed according to one single dimension, typically based on sector, country or factor decompositions. In reality, the risk and performance of a portfolio can be explained by a combination of several such dimensions, and the question therefore arises to assess what the marginal contributions are of various sectors, countries and factors to the performance and risk of a given equity portfolio. In this section, we introduce fundamental betas augmented with sector attributes for multi-dimensional risk and performance analysis which intend to allow asset managers to decompose the risk and performance of a given portfolio across multiple dimensions. For simplicity of exposure, we will focus on two dimensions, namely factor and sectors, but the method presented here can easily be extended to include additional dimensions such as regions. In what follows, we first provide a broad overview of the methodology, before applying it to the equally-weighted portfolio of the S&P 500 universe.

#### 3.1.1 One-Factor Model with Beta as Function of Attributes (Fundamentals and Sectors)

We consider the following one-factor model for stock returns, in which the alpha and the beta are functions both of the sector attribute and of the three observable attributes that were used in Section 2, namely the market capitalisation, the book-to-market ratio and the past one-year return. Hence, we have the following relations:

$$R_{i,t} = \alpha_{i,t} + \beta_{i,t} \times R_{m,t} + \varepsilon_{i,t}$$

### 3. Applications of Fundamental Beta

$$\begin{aligned}\alpha_{i,t} &= \theta_{\alpha,0,s(i)} + \theta_{\alpha,Cap,i} \times Cap_{i,t} + \theta_{\alpha,Bmk,i} \\ &\quad \times Bmk_{i,t} + \theta_{\alpha,Ret,i} \times Ret_{i,t} \\ \beta_{i,t} &= \theta_{\beta,0,s(i)} + \theta_{\beta,Cap,i} \times Cap_{i,t} + \theta_{\beta,Bmk,i} \\ &\quad \times Bmk_{i,t} + \theta_{\beta,Ret,i} \times Ret_{i,t}\end{aligned}\quad (3.1)$$

These quantities are called a "fundamental alpha" and a "fundamental beta". For  $N$  stocks, the model has  $6N+2S$  coefficients (with  $S$  sectors) to estimate. Each stock is attributed to only one sector which is represented with a dummy variable. Sector coefficients  $\theta_{\alpha,0,s(i)}$  and  $\theta_{\beta,0,s(i)}$  are common to stocks that belong to the same sector.

The coefficients are not independent from one stock to the other because stocks from a given sector share the same parameters  $\theta_{\alpha,0,s(i)}$  and  $\theta_{\beta,0,s(i)}$ . The model is estimated by minimising the sum of squared residuals  $\varepsilon_{i,t}$  over all dates and stocks.

$$\text{minimise } \sum_{i,t} \varepsilon_{i,t}^2$$

The conditional beta of a portfolio is the weighted sum of those of the constituents:

$$\begin{aligned}\beta_{p,t} &= \sum_i w_{i,t-1} \beta_{i,t} = \sum_i w_{i,t-1} \theta_{\beta,0,s(i)} + \underbrace{\sum_i w_{i,t-1} \theta_{\beta,Cap,i} Cap_{i,t}}_{=\theta_{\beta,Cap,p} Cap_{p,t}} \\ &\quad + \underbrace{\sum_i w_{i,t-1} \theta_{\beta,Bmk,i} Bmk_{i,t}}_{=\theta_{\beta,Bmk,p} Bmk_{p,t}} \\ &\quad + \underbrace{\sum_i w_{i,t-1} \theta_{\beta,Ret,i} Ret_{i,t}}_{=\theta_{\beta,Ret,p} Ret_{p,t}}\end{aligned}$$

Since each stock belongs to one sector only, we have:

$$\sum_i w_{i,t-1} \theta_{\beta,0,s(i)} = \sum_j w_{t-1}^j \theta_{\beta,0,j},$$

where  $w_{t-1}^j$  denotes the  $j$ -th sector weight in portfolio  $p$ . We can decompose the

expected return of the portfolio  $p$ , conditional on the stocks' weights (actually sector allocation) at date  $t$  in the portfolio:

$$\begin{aligned}E_{t-1}[R_{p,t}] &= \alpha_{i,t-1} + \beta_{i,t-1} \times E_{t-1}[R_{m,t}] \\ &= \alpha_{p,t-1} + \beta_{p,t-1} \times \lambda_{market} \\ &= \left( \sum_j w_{t-1}^j \theta_{\alpha,0,j} + \theta_{\alpha,Cap,p} Cap_{p,t} \right. \\ &\quad \left. + \theta_{\alpha,Bmk,p} Bmk_{p,t} + \theta_{\alpha,Ret,p} Ret_{p,t} \right) \\ &\quad + \left( \sum_j w_{t-1}^j \theta_{\beta,0,j} + \theta_{\beta,Cap,p} Cap_{p,t} \right. \\ &\quad \left. + \theta_{\beta,Bmk,p} Bmk_{p,t} + \theta_{\beta,Ret,p} Ret_{p,t} \right) \lambda_{market}\end{aligned}\quad (3.2)$$

The constant market premium in this equation must be replaced by the conditional market premium in case this parameter is time-varying.

$$\begin{aligned}V_{t-1}[R_{p,t}] &= \mathbf{B}'_{t-1} V_{t-1}[R_{m,t}] \mathbf{B}_{t-1} + \sigma_\varepsilon^2 \\ &= \begin{bmatrix} \sum_j w_{t-1}^j \theta_{\beta,0,j} \\ \theta_{\beta,Cap,p} Cap_{p,t} \\ \theta_{\beta,Bmk,p} Bmk_{p,t} \\ \theta_{\beta,Ret,p} Ret_{p,t} \end{bmatrix} \\ &\quad \times \begin{bmatrix} \sum_j w_{t-1}^j \theta_{\beta,0,j} & \theta_{\beta,Cap,p} Cap_{p,t} \\ \theta_{\beta,Bmk,p} Bmk_{p,t} & \theta_{\beta,Ret,p} Ret_{p,t} \end{bmatrix} \\ &\quad \times V_{t-1}[R_{m,t}] + \sigma_\varepsilon^2 \\ &= \left( \left( \sum_j w_{t-1}^j \theta_{\beta,0,j} \right)^2 + \left( \theta_{\beta,Cap,p} Cap_{p,t} \right)^2 \right. \\ &\quad \left. + \left( \theta_{\beta,Bmk,p} Bmk_{p,t} \right)^2 \right. \\ &\quad \left. + \left( \theta_{\beta,Ret,p} Ret_{p,t} \right)^2 \right) \times \sigma_{market}^2 + \sigma_\varepsilon^2\end{aligned}\quad (3.3)$$

In order to include the country attribute in this decomposition, one needs a version of the fundamental beta and alpha in which these parameters depend on the



### 3. Applications of Fundamental Beta

geographical classification in addition to the sector and the three continuous attributes. This model can be written as:

$$R_{i,t} = \alpha_{i,t} + \beta_{i,t} \times R_{m,t} + \varepsilon_{i,t}$$

$$\alpha_{i,t} = \theta_{\alpha,0,s(i)} + \theta_{\alpha,0,c(i)} + \theta_{\alpha,Cap,i} \times Cap_{i,t} + \theta_{\alpha,Bmk,i} \times Bmk_{i,t} + \theta_{\alpha,Ret,i} \times Ret_{i,t}$$

$$\beta_{i,t} = \theta_{\beta,0,s(i)} + \theta_{\beta,0,c(i)} + \theta_{\beta,Cap,i} \times Cap_{i,t} + \theta_{\beta,Bmk,i} \times Bmk_{i,t} + \theta_{\beta,Ret,i} \times Ret_{i,t}$$

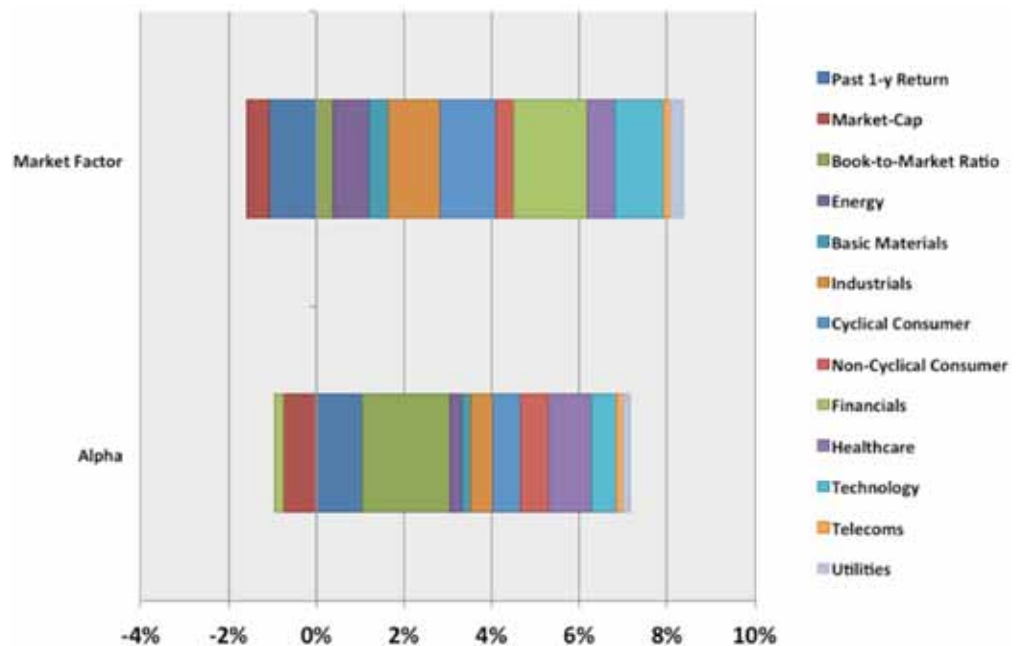
The number of coefficients to estimate now grows to  $6N+2S+2C$  (with  $S$  sectors and  $C$  countries/regions). But the previous regression contains collinear variables because the sum of country dummies variables is constant and equal to 1 for any asset  $i$  and replicates the sum of sectors dummies variables. Therefore, one constraint must be applied to obtain a unique solution. This can be done, for instance, by choosing one country or

one sector as a reference and letting its coefficient be equal to zero. The number of coefficients to estimate is now equal to  $6N+2S+2C-2$ .

#### 3.1.2 Illustration with the US Equally-Weighted Broad Index

The model is estimated over the S&P 500 universe with  $N = 500$  stocks and the period 2002-2015 (which corresponds to 51 quarterly returns). The stocks are classified into the 10 sectors of the TRBC list, which are Energy, Basic Materials, Industrials, Cyclical Consumer, Non-Cyclical Consumer, Financials, Healthcare, Technology, Telecoms and Utilities. As in Section 1, we focus on the decomposition of expected return and volatility of the equally-weighted portfolio of the S&P 500 universe. We also perform the decomposition both for absolute return and risk and for excess return and tracking error with respect to the market factor.

Figure 17: Absolute Performance Decomposition of the Equally-Weighted Portfolio of the S&P 500 Universe on the Market Factor with Fundamental and Sector Attributes  
 The coefficients of the one-factor model are estimated with a pooled regression of the 500 stocks from the S&P 500 universe. Data is quarterly and spans the period 2002-2015, and market returns are from Ken French's library. Attributes (capitalisation, book-to-market and past one-year return) and sector classification come from the ERI Scientific Beta US database and are updated quarterly. We use formula (3.2) to make the performance attribution.



### 3. Applications of Fundamental Beta

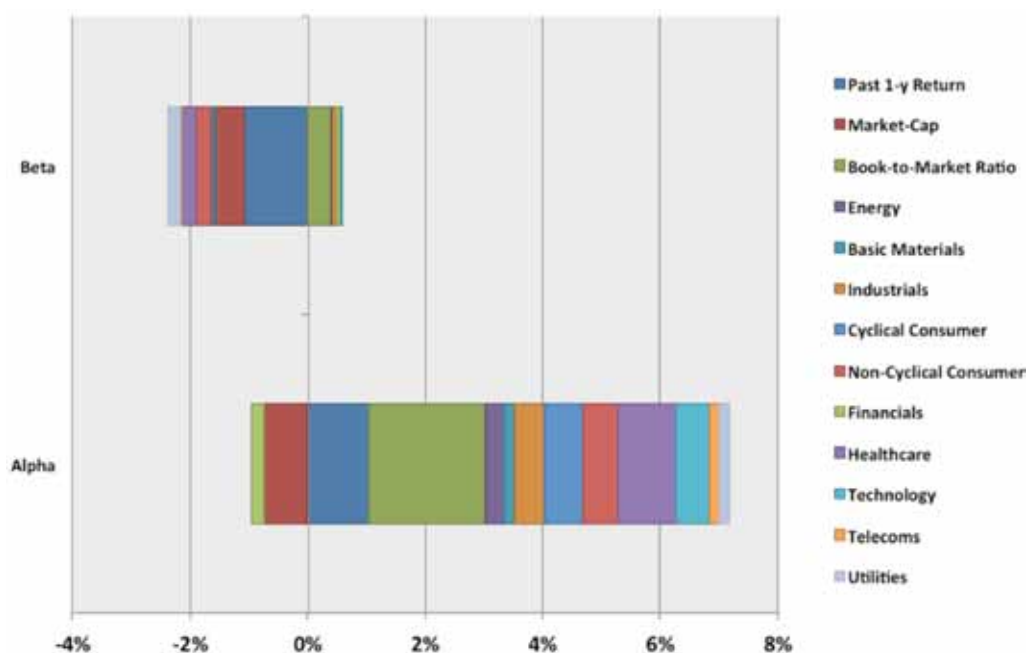
At the first level, there are two components in the expected return: the contribution from the market factor and the alpha. At the second level, we split each of these two elements into fundamental and sector attributes according to the previous model. Figure 17 shows the results. Book-to-market ratio has a positive impact on market exposure and alpha, so that a higher book-to-market ratio implies higher abnormal performance and market exposure. In contrast, the past one-year return has a positive impact on the alpha but a negative impact on the market exposure. Finally the market capitalisation has a negative impact on both alpha and market exposure: the model predicts that other things being equal, large stocks will have smaller abnormal performance and market exposure. In Figure 18, we focus on the excess return, which, as usual, shows a lower market contribution than the absolute performance.

Sector coefficients in Equation (3.1) decompose the intercept coefficients  $\theta_{\alpha,0,i}$  and  $\theta_{\beta,0,i}$  from Equation (2.5). The largest sector contributions within the market component are those of Financials, Industrial and Cyclical Consumer, and Healthcare stands out among the contributions of the various sectors to the abnormal performance.

Figure 19 shows that most of the ex-post risk of portfolio arises from the market risk while the relative risk decomposition in Figure 20 highlights the role of specific risk as being the main contributor to portfolio volatility. For both absolute risk and performance decomposition, Financials, Cyclical consumers and Industrials still appear to be the sectors that contribute most to market exposure.

Figure 18: Relative Performance Decomposition of the Equally-Weighted Portfolio of the S&P 500 Universe on the Market Factor with Fundamental and Sector Attributes (Method 3)

The coefficients of the one-factor model are estimated with a pooled regression of the 500 stocks from the S&P 500 universe. Stocks' returns are in excess of market portfolios returns. Data is quarterly and spans the period 2002-2015, and market returns are from Ken French's library. Attributes (capitalisation, book-to-market and past one-year return) and sector classification come from the ERI Scientific Beta US database and are updated quarterly. We use formula (3.2) to make the performance attribution.



### 3. Applications of Fundamental Beta

Figure 19: Absolute Risk Decomposition Using Euler Decomposition of the Equally-Weighted Portfolio of the S&P 500 Universe on the Market Factor with Fundamental and Sector Attributes (Method 3)

The coefficients of the one-factor model are estimated with a pooled regression of the 500 stocks from the S&P 500 universe. Data is quarterly and spans the period 2002-2015, and market returns are from Ken French's library. Attributes (capitalisation, book-to-market and past one-year return) and sector classification come from the ERI Scientific Beta US database and are updated quarterly. We use formula (3.3) to make the performance attribution.

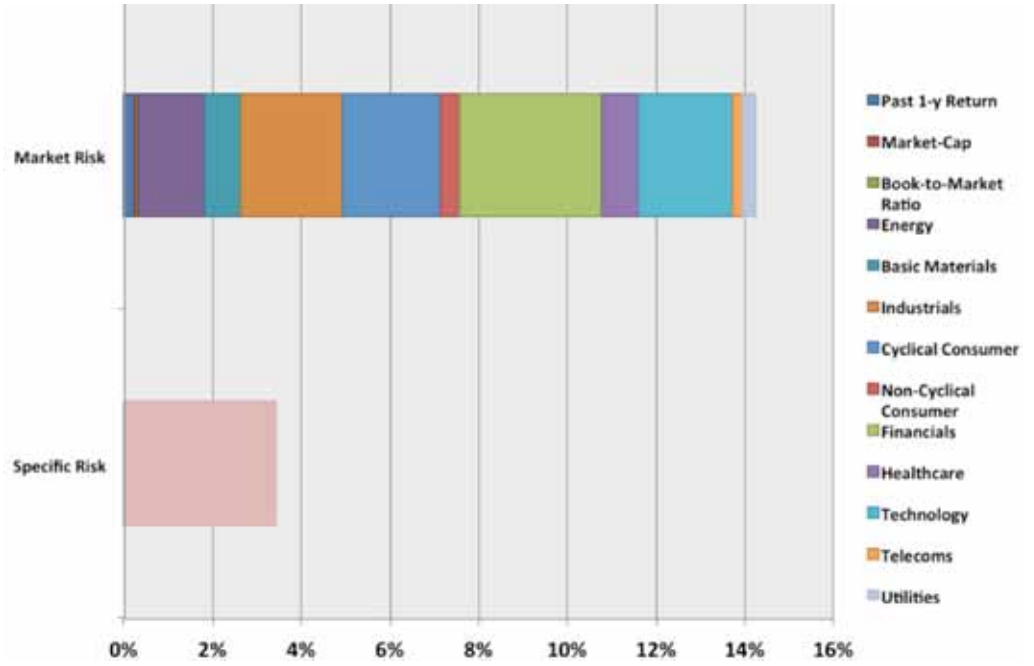
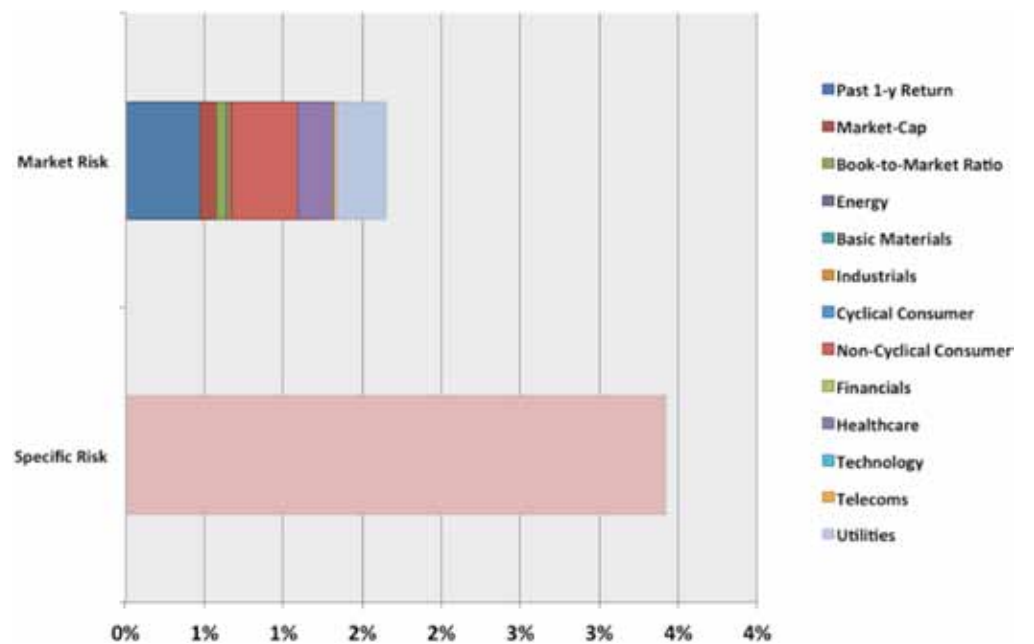


Figure 20: Relative Risk Decomposition Using Euler Decomposition of the Equally-Weighted Portfolio of the S&P 500 Universe on the Market Factor with Fundamental and Sector Attributes (Method 3)

The coefficients of the one-factor model are estimated with a pooled regression of the 500 stocks from the S&P 500 universe. Stocks' returns are in excess of market portfolios returns. Data is quarterly and spans the period 2002-2015, and market returns are from Ken French's library. Attributes (capitalisation, book-to-market and past one-year return) and sector classification come from the ERI Scientific Beta US database and are updated quarterly. We use formula (3.3) to make the risk attribution.



## 3. Applications of Fundamental Beta

### 3.2 Targeting Market Neutrality with Fundamental versus Historical Betas

The construction of a market-neutral portfolio, that is a portfolio with a beta of one, depends inherently on the ability of the portfolio manager to accurately measure the exposure of his portfolio conditional on the current information.<sup>9</sup> The traditional approach to estimating a time-varying beta is to run rolling-window regressions, but it tends to smooth variations over time, thereby slowing down the diffusion of new information in the beta. In contrast, the fundamental beta is an explicit function of the most recent values of the stock's characteristics, and is thus not subject to the same delay issue as the rolling-window one. Our goal in this subsection is to test whether the fundamental beta is a better estimate of the conditional beta by comparing market-neutral portfolios constructed with the two methods.

We focus on the more flexible version of the fundamental beta (see Section 2.3). Each portfolio is a maximum deconcentration subject to the constraint  $\beta_{portfolio} = 1$ . Mathematically, the weights are the solution to the optimisation program:

$$\max_{w_1, \dots, w_N} \frac{1}{\sum_{i=1}^N w_i^2} \quad \text{subject to} \quad \sum_{i=1}^N w_i = 1$$

$$\text{and} \quad \sum_{i=1}^N w_i \beta_i = 1,$$

where the  $\beta_i$  are the constituents' betas, estimated either by the statistical or the fundamental approach. In the absence of beta neutrality constraints, the solution to the optimisation program is an equally-weighted portfolio. Thus, the optimisation program generates

the closest approximation, to a naively diversified equally-weighted portfolio that satisfies the target factor exposure constraint. The portfolio is rebalanced every quarter, with revised estimates for the betas.

It is important to note that the portfolio has by construction a beta of 1 within the estimation period but not necessarily in the backtesting period since realised out-of-sample betas of the constituents are different from the estimated betas. If the true conditional betas were known, the out-of-sample beta of the portfolio measured over a very long period would be equal to 1, because there would be no systematic prediction error, either positive or negative. In reality, the true conditional betas are not observable and are only estimated. The purpose of our comparison is precisely to find which of the two estimation methods yields the best approximation for these unknown parameters.

To avoid look-ahead bias, the coefficients  $\theta$  that relate the fundamental beta to the characteristics are estimated at each rebalancing date over a 5-year rolling window of quarterly data. Historical beta is estimated over the same sample. In order to achieve more robustness in the results, we do not conduct the comparison for a single universe, but we instead repeat it for 1,000 random universes of 30 stocks picked among the 218 that remained in the S&P 500 universe between 2002 and 2015. Hence we have 1,000 random baskets of 30 stocks, and, for each basket, we compute the two market-neutral portfolios.

In order to test whether the two portfolios achieve the neutrality target, we regress their returns against the market on the

<sup>9</sup> - In a long-short context, market neutrality rather refers to a portfolio with a beta of 0. Here we focus on a long-only context, where beta neutrality is used to refer to a portfolio with a beta of 1.

### 3. Applications of Fundamental Beta

13-year period. The resulting out-of-sample market beta is taken as a measure of the ex-post market neutrality. We complete this indicator with the market correlation computed over the period 2002-2015. Table 3 shows that portfolios based on fundamental beta achieve, on average, better market neutrality than those based on time-varying historical beta, with an in-sample beta of 0.925 versus 0.869, on average, across the 1,000 universes. We observe the same phenomenon in term of correlation with an average market correlation of 0.914 for portfolios based on fundamental betas, versus 0.862 for the portfolios based on historical time-varying beta. Since these numbers are only averages across the 1,000 universes, we also compute the standard deviations of the 1,000 out-of-sample betas or correlations around their respective means. The dispersion levels are close for both methods.

To check whether these results are robust to the choice of the sample period, we perform again the comparison between the two methods on a longer sample, which spans the period 1970-2015. The results from Table 4 are clear: the portfolios constructed with the fundamental method are ex-post closer to neutrality than those that rely on rolling-window betas. We take advantage of the longer sample size to compute the out-of-sample beta on a

5-year rolling window. The first five years of the sample are used to calibrate the initial betas. Thus, the first beta is available in December 1979, that is ten years after the beginning of the sample. Over the 44-year sample, both betas move around their means, as shown in Figure 21. The average beta of the portfolios constructed with the fundamental approach is not systematically closer to 1 than that of the portfolios based on the traditional historical approach, but it exhibits less time variation. In particular, the average beta of the latter portfolios over the period March 1991-March 1996 is as low as 0.37, a number that indicates a severe deviation from neutrality. In September 2008, the ex-post average is at 1.18, which means that on average, the historical method led to portfolios that were more aggressive than expected between 2003 and 2008. With the fundamental betas, the range of ex-post betas is narrower, between 0.74 and 1.15.

Table 3: Targeting Beta Neutrality for Maximum Deconcentration Portfolios Based on Fundamental and Time-Varying Historical Betas (2002-2015)

1,000 maximum deconcentration portfolios of 30 random stocks subject to a beta neutrality constraint are constructed by using the rolling-window or the fundamental betas. The 30 stocks are picked among the 218 that remained in the S&P 500 universe for the period 2002-2015, and the portfolios are rebalanced every quarter. The control regression on Ken French's market factor is done using quarterly returns over the period 2002-2015. The market beta and the market correlation are computed for each portfolio over the period 2002-2015 and are averaged across the 1,000 universes. Also reported are the standard deviations of the beta and the correlation over the 1,000 universes.

	Out-of-Sample Market beta		Out-of-Sample Market correlation	
	Mean	Standard Deviation	Mean	Standard Deviation
Historical	0.869	0.032	0.862	0.025
Fundamental	0.925	0.035	0.914	0.020

### 3. Applications of Fundamental Beta

Figure 21: Mean Ex-Post Betas (Estimated on 5-Year Rolling Window) Over 1,000 Universes

1,000 maximum deconcentration portfolios of 30 random stocks subject to a beta neutrality constraint are constructed by using the rolling-window or the fundamental betas. The 30 stocks are picked among the 71 that remained in the S&P 500 universe for the period 1970-2015, and the portfolios are rebalanced every quarter. The control regression on Ken French's market factor is done on a 5-year rolling window of quarterly returns. For each window, the market beta is averaged across the 1,000 universes.

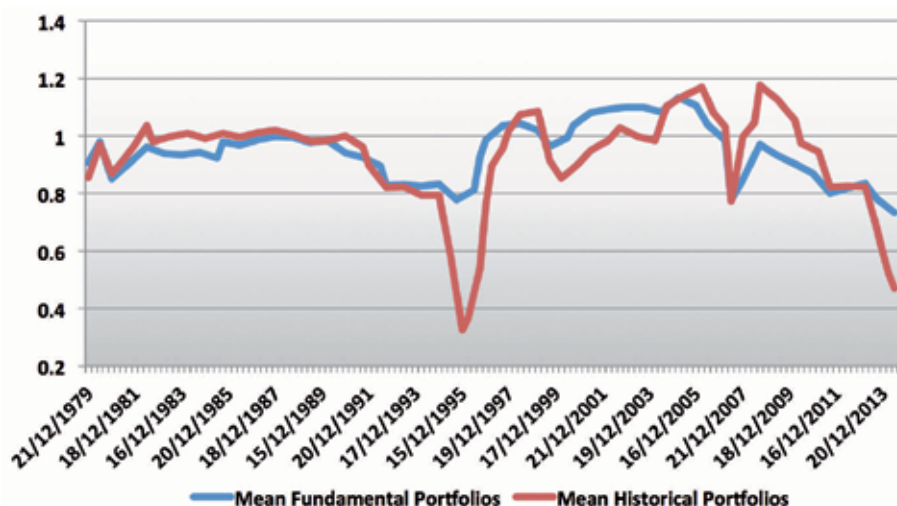


Table 4: Targeting Beta Neutrality for Maximum Deconcentration Portfolios Based on Fundamental and Time-Varying Historical Betas (1970-2015)

1,000 maximum deconcentration portfolios of 30 random stocks subject to a beta neutrality constraint are constructed by using the rolling-window or the fundamental betas. The 30 stocks are picked among the 71 that remained in the S&P 500 universe for the period 1970-2015, and the portfolios are rebalanced every quarter. The control regression on Ken French's market factor is done using quarterly returns over the period 1970-2015. The market beta and the market correlation are computed for each portfolio over the period 1970-2015 and are averaged across the 1,000 universes. Also reported are the standard deviations of the beta and the correlation over the 1,000 universes.

	Out-of-Sample Market beta		Out-of-Sample Market correlation	
	Mean	Standard Deviation	Mean	Standard Deviation
Historical	0.931	0.019	0.845	0.021
Fundamental	0.950	0.015	0.910	0.010

It can be argued that Figure 21 focuses only on the average situation and, as such, hides differences across the 1,000 universes. Thus, we look at the "worst" of the universes in Figure 22. At each date, the 1,000 absolute differences between the 5-year rolling-window beta and the target of 1 are computed, and the figure shows the highest value. More often than not, it is with the historical method that the largest deviation is observed. There are a number of months (March 1996, December 2005 and March 2007 being the most extreme examples) where the relative error exceeds 60%.

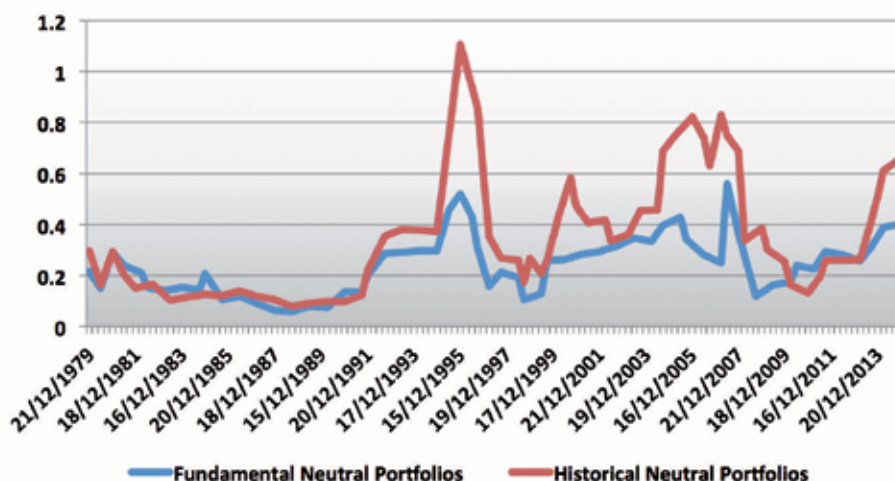
Eventually, the fundamental method appears to be a more reliable way of constructing market-neutral portfolios. The fact that it leads to portfolios that are ex-post more neutral suggests that it allows the prediction error in the estimation of the conditional betas to be reduced. In other words, it approximates the true conditional beta better than the classical rolling-window method does.



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Figure 22: Largest Distance to 1 across 1,000 Universes

1,000 maximum deconcentration portfolios of 30 random stocks subject to a beta neutrality constraint are constructed by using the rolling-window or the fundamental betas. The 30 stocks are picked among the 71 that remained in the S&P 500 universe for the period 1970-2015, and the portfolios are rebalanced every quarter. The control regression on Ken French's market factor is done on a 5-year rolling window of quarterly returns. For each window, the figure shows the largest distance to 1 computed over the 1,000 universes.



#### 3.3 Asset Pricing with the Fundamental CAPM

The goal of an asset pricing model is to explain the differences in expected returns across assets through the differences in their exposures to a set of pricing factors. It is well known that the standard CAPM largely misses this goal, given its inability to explain effects such as size, value and momentum. In this subsection, we ask whether the fundamental CAPM introduced in Section 2.3 is more successful from this perspective. To this end, we conduct formal asset pricing tests by using Fama and MacBeth method (1973). There are two statistics of interest in the output of these tests. The first one is the average alpha of the test portfolios, which measures the fraction of the expected return that is not explained by the model. The second set of indicators is the set of factor premia estimates, which should have plausible values.

##### 3.3.1 Fundamental CAPM with Constant Market Risk Premium

###### Unconditional Form of the Model

In the conditional CAPM, the expected return of a stock  $i$  conditional on the information available at date  $t$  is a linear function of the stock's conditional beta, the slope coefficient being the conditional market premium. Mathematically, this is written as:

$$E_{t-1}[R_{i,t}] = \beta_{i,t-1}E_{t-1}[Rm_t]. \quad (3.4)$$

In this model, expected returns can vary both in the cross-section and the time-series: cross-sectional variation is generated by the factor exposure only, while time variation results from changes in the market premium or the beta. We first consider the case where the conditional market premium is constant, and we relax this assumption in Section 3.3.2.

Since conditional expected returns are not observable, we follow Jagannathan and Wang (1996) in transforming the



### 3. Applications of Fundamental Beta

conditional model into an unconditional one. To do this, we take expectations in both sides of (3.4) to obtain:

$$E[R_{i,t}] = E[\beta_{i,t-1}]E[Rm_t]. \quad (3.5)$$

In the static CAPM, this equation is replaced by:

$$E[R_{i,t}] = \beta_i E[Rm_t], \quad (3.6)$$

where  $\beta_i$  is the unconditional beta. (3.5) and (3.6) are not equivalent: in the conditional model, what determines the unconditional expected return of a stock is its average conditional beta, rather than its unconditional beta. In fact, as shown in Appendix A1, the two quantities would be equal if both the conditional market premium and the conditional market variance were constant, but although we assume here that market returns have a constant first moment, we do not assume that they are homoskedastic (i.e. that they have a constant variance).

#### Estimation and Measurement of Pricing Errors

Consider the following one-factor model associated with the fundamental CAPM. For each test portfolio, we have:

$$R_{it} = \alpha_i + \beta_{i,t-1}Rm_t + \varepsilon_{it},$$

where  $\beta_{i,t-1}$  is the fundamental beta and the residuals are centred and uncorrelated from the market. Taking conditional expectations in both sides of this equation, we obtain:

$$\begin{aligned} E_{t-1}[R_{it}] &= \alpha_i + E_{t-1}[\beta_{i,t-1}Rm_t] \\ &= \alpha_i + \beta_{i,t-1}E_{t-1}[Rm_t]. \end{aligned}$$

Hence, Equation (3.4) holds if, and only if,  $\alpha_i = 0$ . Thus, we are interested in testing whether  $\alpha_i = 0$ . Second, we also want to estimate the unconditional market premium implied by the model in order to check that it is plausible in

view of the observed market returns. We do this by using the Fama and MacBeth (1973) procedure. We also apply it to the estimation of the alpha in the static CAPM, in which the beta is constant over time.

In the static CAPM, the first step of Fama-MacBeth approach is to estimate the beta of each portfolio through a time-series regression:

$$\begin{aligned} R_{it} &= c_i + \hat{\beta}_i Rm_t + u_{it}, \\ t &= 1, \dots, T. \end{aligned}$$

In the fundamental CAPM, this step is bypassed by estimating a time-varying beta along the lines of the procedure described in Section 2.3.

In the static CAPM, the second step is to run a cross-sectional regression of stock returns on the estimated betas at each date, so as to obtain a time-series of estimates for the market premium:

$$\begin{aligned} R_{i,t} &= \hat{d}_t + \hat{\lambda}_t \times \hat{\beta}_i + v_{it}, \\ i &= 1, \dots, N. \end{aligned} \quad (3.7)$$

In the fundamental CAPM, this step is modified by regressing the cross-section of stock returns on the time-series average of the conditional betas. Taking the average of conditional betas is consistent with Equation (3.5), where it is the expected conditional beta that determines the unconditional expected return of a stock.

The last step is to form estimators for the market premium and for the alpha of each portfolio. The estimators have the same form in both models: they are equal to the averages of the cross-sectional regression estimates:

$$\hat{\lambda} = \frac{1}{T} \sum_{t=1}^T \hat{\lambda}_t \quad \text{and} \quad \hat{\alpha}_i = \frac{1}{T} \sum_{t=1}^T (\hat{d}_t + v_{it}).$$

### 3. Applications of Fundamental Beta

If the model is correctly specified,  $\hat{\alpha}_i$  should not be significantly different from zero, and  $\hat{\lambda}$  should be close to the average market return. In order to test these restrictions, we need to estimate the standard deviations of the estimators. Simple estimates are given by:<sup>10</sup>

$$\sigma^2(\hat{\lambda}) = \frac{1}{T^2} \sum_{t=1}^T (\hat{\lambda}_t - \hat{\lambda})^2$$

$$\text{and } \sigma^2(\hat{\alpha}_i) = \frac{1}{T^2} \sum_{t=1}^T (d_t + v_{it} - \hat{\alpha}_i)^2.$$

It is also interesting to compare the fundamental CAPM with a multi-factor model in which stocks' attributes are used to construct additional factors instead of being taken as instrumental variables for the estimation of the beta. The multi-factor model that corresponds to our choice of characteristics is the Carhart four-factor model, in which expected returns are given by:

$$E[R_{i,t}] = \lambda \times \beta_i + \lambda_{size} \times \beta_i^{size} + \lambda_{value} \times \beta_i^{value} + \lambda_{mom} \times \beta_i^{mom}.$$

( $\lambda$  and  $\beta_i$  denote respectively the market premium and the market beta.) The Fama-MacBeth procedure can be immediately extended to this multi-factor framework, and it yields estimates for the alphas of the test portfolios as well as for the four risk premia.

#### Test Assets and Results

Our test assets are the 30 decile portfolios formed by sorting stocks on size, book-to-market or past one-year return. We consider the stocks of the S&P 500 universe, and we construct equally-weighted portfolios for which we measure quarterly returns over the period 1973–2014. Appendix A2 provides descriptive statistics on these portfolios: we verify that there is a value, a size and a

momentum effect in this sample, at least between the two extreme sides of the classification.

For each model, Table 6 reports the average alpha across the 30 portfolios, as well as the risk premia estimated by the Fama-MacBeth procedure. These premia are to be compared with the historical average returns to the factors, which are displayed in Table 5. In Table 6, the static CAPM gives the largest average alpha, at 5.04%. Table 7 provides more information about the dispersion of alphas across test portfolios by showing the 25th, 50th and 75th percentiles of the distribution: the static CAPM has the largest quantiles, and 25% of the portfolios have alphas greater than 6.35%. Moreover, alphas tend to be more statistically significant for this model than for the other two, as appears from the t-statistics. On the other hand, the Carhart model and the conditional CAPM have comparable average alphas, of 2.87% and 2.86% respectively. From Table 7, the conditional CAPM is the one that can achieve the smallest pricing errors, since it has the lowest first quartile. For both models, alphas are, on average, insignificant, as can be seen from the low t-statistics. These results confirm the results from the literature, saying that the return spreads between portfolios sorted on size, value or short-term past return are not explained by the standard CAPM. But they show that from a statistical standpoint, a model in which the characteristics are used as drivers of factor exposures performs as well as one in which the characteristics are used as sorting criteria to define additional factors. In other words, the differences across the average returns to size, value and momentum portfolios can be as well explained with the fundamental CAPM as with a multi-factor model.

10 - As pointed by Jagannathan and Wang (1996), these estimates do not take into account the sampling errors in estimated betas.

### 3. Applications of Fundamental Beta

Across the three models studied here, it is the fundamental CAPM that delivers the market premium estimate (7.14%) that is the closest to the sample average (6.88%). The other two models fall short of the historical mean, with estimates of about 4.5%. For the Carhart model, the premia estimated by Fama-MacBeth regressions have reasonable values, though they do not align well with the realised premia of Table 5.

In closing, these tests raise less evidence against the fundamental CAPM than against the static CAPM, and they raise

no more evidence against it than against the four-factor model. Hence, while these tests lead to reject the static CAPM, they do not allow us to reject the fundamental CAPM and the four-factor model, and the two models have comparable success in explaining the returns to the test assets.

#### 3.3.2 Fundamental CAPM with Time-Varying Market Risk Premium

The expression of the conditional CAPM in Equation (3.4) naturally leads to the introduction of the conditional market premium. In this subsection, we relax the assumption that the conditional premium

Table 5: Annualised Ex-Post Risk Premia Over 1973-2014

Market	Size	Value	Momentum
6.88%	2.80%	4.48%	8.20%

We compute average of factor annualised returns with quarterly data from Ken French's library over the period 1973-2014.

Table 6: Average Alphas and Estimated Factor Risk Premia From Fama-MacBeth Regressions

	Average Alpha	Market Premium	Size Premium	Value Premium	Momentum Premium
Static CAPM	5.04% (1.66)	4.54% (0.76)			
Carhart	2.87% (0.89)	4.07% (0.74)	5.85% (8.36)	8.01% (9.65)	6.48% (4.91)
Fundamental CAPM	2.86% (0.79)	7.14% (0.90)			

For each of the three pricing models, we perform Fama-MacBeth regressions of the returns to 30 portfolios sorted on size, book-to-market or past one-year return on the market factor (for the static and the fundamental CAPM) or the market augmented with the size, value and momentum factors (for the Carhart model). The difference between the static and the fundamental CAPM is that in the former model, a single beta is estimated for each test portfolio, while in the latter, each portfolio has a time-varying beta that is a function of the attributes of the constituents. The test portfolios are equally weighted and their constituents are the 500 stocks from the S&P 500 universe. Regressions are done on the period 1973-2014. The first column contains the average alpha across the 30 portfolios, and the next columns display the estimated risk premium of each factor included in the model. The numbers in brackets in the first column are the average t-statistics estimated by the Fama-MacBeth method. In the other columns, they are t-statistics for the risk premia estimates.

Table 7: Distribution of Estimated Alphas over the Cross-Section of Sorted Portfolios

	Mean	Standard Deviation	First Quartile	Median	Third Quartile
Static CAPM	5.04%	2.74%	3.35%	4.95%	6.35%
Carhart	2.87%	1.06%	2.35%	2.66%	3.57%
Fundamental CAPM	2.86%	2.72%	1.08%	2.76%	4.19%

For each of the three pricing models, we perform Fama-MacBeth regressions of the returns to 30 portfolios sorted on size, book-to-market or past one-year return on the market factor (for the static and the fundamental CAPM) or the market augmented with the size, value and momentum factors (for the Carhart model). The difference between the static and the fundamental CAPM is that in the former model, a single beta is estimated for each test portfolio, while in the latter, each portfolio has a time-varying beta that is a function of the attributes of the constituents. The test portfolios are equally weighted and their constituents are the 500 stocks from the S&P 500 universe. Regressions are done on the period 1973-2014. This table shows the mean, the standard deviation and the 25th, the 50th and the 75th percentiles of the distribution of alphas across the 30 test portfolios.

### 3. Applications of Fundamental Beta

is constant. This is more realistic, as it is well documented that some variables, including notably the dividend yield and the default spread, have power to predict stock returns, at least at long horizons (Fama and French, 1988, 1989; Hodrick, 1992; Menzly, Santos and Veronesi, 2004). As Jagannathan and Wang (1996) show, introducing a time-varying market premium implies that the unconditional expected return of a stock depends on its average conditional beta – as it already does in the conditional CAPM of Section 3.3.1 – but also on the covariance between the conditional beta and the conditional market premium. We thus want to test whether this covariance term improves the ability of the fundamental CAPM to explain the returns of portfolios sorted on size, book-to-market or short-term past return with respect to the case where the premium is constant.

#### Derivation of Unconditional Model

When the conditional market risk premium,  $\lambda_t$ , varies over the business cycle, conditional expected stock returns are given by (see Equation (3.4)):

$$E_{t-1}[R_{i,t}] = \beta_{i,t-1}\lambda_{t-1}.$$

Taking expectations in both sides, Jagannathan and Wang (1996) obtain the following model for unconditional expected returns:

$$E[R_{i,t}] = \bar{\lambda}\bar{\beta}_i + Cov(\lambda_{t-1}, \beta_{i,t-1}), \quad (3.8)$$

where

$$\bar{\lambda} = E[\lambda_{t-1}], \quad \bar{\beta}_i = E[\beta_{i,t-1}].$$

Here  $\bar{\lambda}$  is the expected market risk premium, and  $\bar{\beta}_i$  is the expected conditional beta. The model in (3.8) reduces to the one in Equation (3.5) if the market premium is uncorrelated from the beta.

As a result, (3.8) can be rewritten as:

$$E[R_{i,t}] = \bar{\lambda}\bar{\beta}_i + c_{prem}\theta_i^{prem}, \quad (3.9)$$

where the "beta-prem sensitivity" of an asset is defined as the sensitivity of its beta with respect to the predictive variable:

$$\theta_i^{prem} = \frac{Cov(\beta_{i,t-1}, \lambda_{t-1})}{Var(\lambda_{t-1})},$$

and the coefficient  $c_{prem}$  is given by:

$$c_{prem} = Var(\lambda_{t-1}).$$

Equation (3.9) says that the unconditional expected return on a stock is a linear function of its expected beta and its beta-prem. Ex-ante, stocks with higher average market exposures earn higher returns, which is the same intuition as in the static CAPM. But stocks that are more exposed to the market when the expected market return is higher, i.e. stocks that have a higher beta-prem, earn also higher expected returns: indeed, a large covariance between the conditional beta and the market premium makes a stock be like a "market follower", and this increased market exposure requires compensation for bearing systematic risk. The coefficient  $c_{prem}$  is the marginal premium earned for an additional unit of covariance between the beta and the market premium.

To model the time-varying market premium, we follow Jagannathan and Wang (1996) in relating the expected market return to the yield spread between BAA and AAA bonds. This choice is motivated by the literature that has linked expected stock returns to the business cycle (Keim and Stambaugh, 1986; Fama and French, 1989), and the work of Stock and Watson (1989), which finds that the default spread is a good predictor of market conditions.

### 3. Applications of Fundamental Beta

Hence, we assume that  $\lambda_t$  is an affine function of the spread:

$$\lambda_t = K_0 + K_1 sp_t.$$

This implies that:<sup>11</sup>

$$E[R_{i,t}] = \bar{\lambda}\bar{\beta}_i + c_{sp}\theta_i^{sp}, \quad (3.10)$$

where:

$$c_{sp} = K_1 Var(sp_{t-1}),$$

$$\theta_i^{sp} = \frac{Cov(\beta_{i,t-1}, sp_{t-1})}{Var(sp_{t-1})}.$$

The coefficient  $\theta_i^{sp}$  is the beta of the conditional beta with respect to the default spread, and  $c_{sp}$  is the sensitivity of the expected return of a stock with respect to its exposure to changes in the default spread.

#### Model Estimation and Tests

The conditional CAPM holds if, and only if, the  $\alpha_i$  in the following one-factor model is zero for each stock  $i$ :

$$R_{i,t} = \alpha_i + \beta_{i,t-1}Rm_t + \varepsilon_{it},$$

We again use Fama-MacBeth procedure. In the first step, we estimate the time-series of fundamental betas for each stock, and we compute the beta-prem sensitivity in Equation (3.10). In the second step, we regress at each date the cross-section of stock returns on the average fundamental beta and the beta-prem sensitivity, in line with Equation (3.9):

$$R_{i,t} = \hat{a}_t + \hat{\lambda}_t \times \bar{\beta}_i + \hat{c}_{sp,t} \hat{\theta}_i^{sp} + v_{it},$$

$$i = 1, \dots, N. \quad (3.11)$$

By averaging the estimates  $\hat{\lambda}_t$  and  $\hat{c}_{sp,t}$  over time, we obtain estimates  $\hat{\lambda}$  and  $\hat{c}_{sp}$  for the average market premium and the coefficient  $c_{sp}$ .

$$\hat{\lambda} = \frac{1}{T} \sum_{t=1}^T \hat{\lambda}_t,$$

$$\hat{c}_{sp} = \frac{1}{T} \sum_{t=1}^T \hat{c}_{sp,t},$$

$$\text{and } \hat{\alpha}_i = \frac{1}{T} \sum_{t=1}^T (\hat{a}_t + v_{it}).$$

Estimates for the average alpha and the average market premium are shown in Table 9. Interestingly, the average alpha is smaller than for the three alternative pricing models considered in Section 3.3.1 (see Table 6): the average alpha is reduced by almost half (from 2.86% to 1.69%) with respect to the fundamental CAPM with a constant market premium. The estimated average market premium is greater than before (it grows from 7.14% to 8.59%) but it remains at a reasonable level. Overall, the introduction of a time-varying premium helps explain the cross-section of expected returns.

Table 11 compares the distributions of alphas across the 30 portfolios for the four competing models. The distribution for the conditional CAPM with time-varying market premium shifts to the left and is closer to 0 than for the model with a constant premium. Furthermore, alpha becomes less significant in the fundamental CAPM with a time-varying premium.

11 - Note that Jagannathan and Wang (1996) use a different form for the unconditional model corresponding to the conditional model of Equation (3.4). Starting from (3.10), they write the unconditional expected return as a function of the unconditional market beta and the beta with respect to the default spread. We work directly with (3.10) because we have an explicit model for the conditional beta (as a function of the stock's attributes), which allows for a direct estimation of the sensitivity of the beta with respect to the predictive variable.

### 3. Applications of Fundamental Beta

*Table 9: Estimated Expected Market Risk Premia and Average Estimated Alphas over the Cross-Section of Stocks (1973-2015)*  
 We first estimate the fundamental betas of 30 equally-weighted portfolios sorted on size, book-to-market or past one-year return. The constituents of these portfolios are picked from the S&P 500 universe, and the fundamental beta of a portfolio is a function of the constituents' attributes. Second, we estimate the beta of each fundamental beta with respect to the yield spread between BAA- and AAA-rated Barclays US bonds, downloaded from Datastream. Regressions are done with quarterly observations over the period 1973-2015. Third, we perform at each date a cross-sectional regression of returns on the average fundamental betas and the yield spread beta, to obtain an estimate for the alpha, the market premium estimate and the coefficient  $c_{sp}$  at each date. The first column in the table reports the average alpha from these regressions, across all dates and test portfolios. The second and the third columns are the average market premium estimate and the average  $c_{sp}$  across all dates.

	Average Alpha	Average Market Premium	$\hat{c}_{sp,t}$
Fundamental CAPM With Time-Varying Market Premium	1.69% (0.71)	8.59% (1.60)	-8.33% (-0.36)

*Table 11: Alphas Distribution over the Cross-Section of Sorted Portfolios*

The first three rows of the table are a reminder of Table 7. They summarise the distribution of the estimated alphas for 30 portfolios sorted on size, book-to-market or past one-year return. These alphas are obtained by performing Fama-MacBeth regressions for three pricing models. The fourth row shows the distribution of alphas obtained in the conditional CAPM with fundamental beta and a time-varying market premium. The fundamental beta is a function of the constituents' attributes. Regressions are done on the period 1973-2015. The last column shows the average t-statistics across alphas.

	Mean	Std	1st Quartile	Median	Third Quartile	Mean Corrected T-stats
Static CAPM	5.04%	2.74%	3.35%	4.95%	6.35%	1.66
Carhart Model	2.87%	1.06%	2.35%	2.66%	3.57%	0.89
Fundamental CAPM	2.86%	2.72%	1.08%	2.76%	4.19%	0.79
Fundamental CAPM with Time-Varying Market Factor	1.69%	2.70%	-0.27%	1.54%	3.39%	0.71

## 4. Conclusion





## 4. Conclusion

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Multi-factor models are standard tools for analysing the performance and the risk of equity portfolios. In the Fama-French and Carhart models, the size, value and momentum factors are constructed by first sorting stocks on an attribute (market capitalisation, the book-to-market ratio or past short-term return), then by taking the excess return of a long leg over a short leg. These models do a much better job than the standard capital asset pricing model (CAPM) at explaining the differences across expected returns. However, numerous patterns have been identified in stock returns, raising concerns about a potential inflation in the number of long-short factors and their overlap. As noted by Cochrane (2001, p. 1060), one must ask "which characteristics really provide *independent* information about average returns" and "which are subsumed by others".

Our work suggests another meaningful approach for explaining the cross-section of expected returns, which consists in treating attributes of stocks as instrumental variables to estimate the beta with respect to the market factor. We stay with a limited number of risk factors by considering a one-factor model, and we estimate a conditional beta that depends on the same three characteristics that define the Fama-French and Carhart factors. We show that a conditional CAPM based on this "fundamental" beta can capture the size, value and momentum effects as well as the Carhart model, but without the help of additional factors. The pricing errors are further reduced by introducing a time-varying market premium, which introduces the cyclical covariation between fundamental beta and the market risk premium as a driver of expected returns. Moreover, we use the fundamental beta approach to embed the sector dimension in our multi-factor risk and performance

analysis. We let the alpha and the market exposure depend on both the sector attribute and the three observable attributes that define the Fama-French-Carhart factors: market capitalisation, the book-to-market ratio and past one-year return. We show that the fundamental beta approach is more convenient when the multi-factor analysis is extended to additional dimensions (e.g. sector and regions). Finally, the fundamental beta provides an alternative measure of the conditional beta, which is a function of observable variables and is not subject to the lag issue that potentially affects betas estimated by a rolling-window regression. It immediately responds to changes in a stock's attributes, which allows us to assess the impact of a change in the portfolio composition on the factor exposure. We illustrate these benefits by constructing market-neutral portfolios based on the fundamental and the rolling-window methods, and we show that the former achieves better out-of-sample neutrality. We do not claim that a one-factor model with time-varying beta is the key to explaining any difference in expected returns. The two approaches – multi-factor model and conditional single-factor model – are not exclusive, and the true (still unknown) asset pricing model is likely to be a multi-factor one with betas depending on state variables.

Our work can be extended in several dimensions. First, one may try to use attributes to decompose exposures to other risk factors with a conditional multi-factor model such as the Fama-French factor model. This requires the identification of the meaningful attributes for each factor. Another possible avenue for further research would consist in extending the empirical analysis by using macroeconomic variables (as in Ferson and Schadt, 1996) in addition to characteristics.

## 4. Conclusion

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Finally, focusing on conditional multi-factor models with time-varying risk exposures allows time-varying risk premia to be considered and introduces cyclical covariation terms between fundamental betas and the associated factor risk premia. This last issue could re-launch the discussion about the study of the cross-section of expected returns. We leave these questions for further research.

## 4. Conclusion

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# Appendix



# Appendix

## A.1 Sufficient Condition for Equality between Average Conditional Beta and Unconditional Beta

Suppose that the conditional expected return and the conditional variance of the market are constant. Then, the unconditional moments equal the conditional ones:

$$\text{Var}[R_{m,t}|\Phi_{t-1}] = \text{Var}[R_{m,t}]$$

$$E[R_{m,t}|\Phi_{t-1}] = E[R_{m,t}]$$

Next, we take the expectation of the conditional beta and we use the fact that conditional moments are constant:

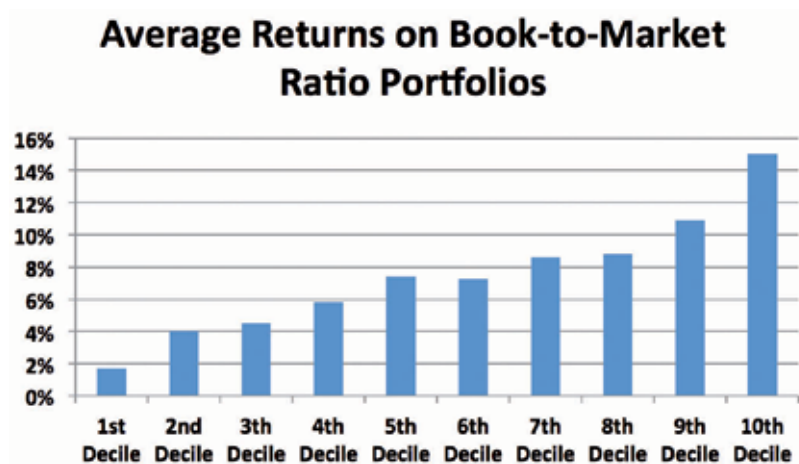
$$\begin{aligned} E[\beta_{i,t-1}^m] &= E\left[\frac{\text{Cov}[R_{i,t}, R_{m,t}|\Phi_{t-1}]}{\text{Var}[R_{m,t}|\Phi_{t-1}]}\right] = E\left[\frac{E[R_{i,t} \times R_{m,t}|\Phi_{t-1}] - E[R_{m,t}|\Phi_{t-1}]E[R_{i,t}|\Phi_{t-1}]}{\text{Var}[R_{m,t}]}\right] \\ &= \frac{E[E[R_{i,t} \times R_{m,t}|\Phi_{t-1}]] - E[E[R_{m,t}]\times E[R_{i,t}|\Phi_{t-1}]]}{\text{Var}[R_{m,t}]} \\ &= \frac{E[R_{i,t} \times R_{m,t}] - E[R_{m,t}]\times E[R_{i,t}]}{\text{Var}[R_{m,t}]} = \frac{\text{Cov}[R_{i,t}, R_{m,t}]}{\text{Var}[R_{m,t}]}, \end{aligned}$$

which is the unconditional beta.

## A.2 Returns on Attribute-Sorted Portfolios (1973–2014)

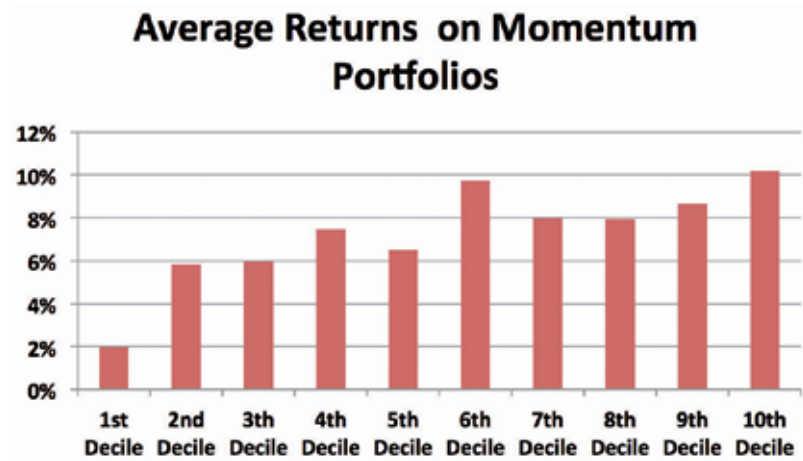
We first form decile portfolios from the S&P 500 universe by sorting stocks on the book-to-market ratio z-score, market-cap z-score and momentum z-score over the period 1973–2014 with quarterly returns. Each portfolio contains 50 stocks in each period.

Portfolio 10 (resp. 1) has the highest (resp. lowest) B/M ratio and the highest (resp. lowest) excess return over the period.

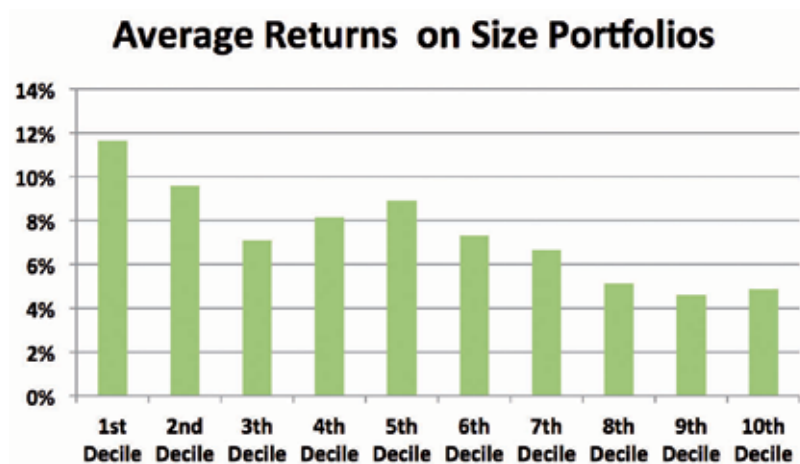


Portfolio 10 (resp. 1) has the highest (resp. lowest) Momentum price and the highest (resp. smallest) excess return over the period.

# Appendix



Portfolio 10 (resp. 1) has the highest (resp. smallest) market cap and the second lowest (resp. highest) excess return over the period



# Appendix

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# About EDHEC-Risk Institute



# About EDHEC-Risk Institute

Founded in 1906, EDHEC is one of the foremost international business schools. Accredited by the three main international academic organisations, EQUIS, AACSB, and Association of MBAs, EDHEC has for a number of years been pursuing a strategy of international excellence that led it to set up EDHEC-Risk Institute in 2001. This institute now boasts a team of close to 50 permanent professors, engineers and support staff, as well as 38 research associates from the financial industry and affiliate professors.

## The Choice of Asset Allocation and Risk Management and the Need for Investment Solutions

EDHEC-Risk has structured all of its research work around asset allocation and risk management. This strategic choice is applied to all of the Institute's research programmes, whether they involve proposing new methods of strategic allocation, which integrate the alternative class; taking extreme risks into account in portfolio construction; studying the usefulness of derivatives in implementing asset-liability management approaches; or orienting the concept of dynamic "core-satellite" investment management in the framework of absolute return or target-date funds. EDHEC-Risk Institute has also developed an ambitious portfolio of research and educational initiatives in the domain of investment solutions for institutional and individual investors.

## Academic Excellence and Industry Relevance

In an attempt to ensure that the research it carries out is truly applicable, EDHEC has implemented a dual validation system for the work of EDHEC-Risk. All research work must be part of a research programme, the relevance and goals of which have been validated from both an academic and a business viewpoint by the Institute's advisory board. This board is made up of internationally recognised researchers, the Institute's business partners, and representatives of major international institutional investors. Management of the research programmes respects a rigorous validation process, which guarantees the scientific quality and the operational usefulness of the programmes.

Six research programmes have been conducted by the centre to date:

- Asset allocation and alternative diversification
- Performance and risk reporting
- Indices and benchmarking
- Non-financial risks, regulation and innovations
- Asset allocation and derivative instruments
- ALM and asset allocation solutions

These programmes receive the support of a large number of financial companies. The results of the research programmes are disseminated through the EDHEC-Risk locations in Singapore, which was established at the invitation of the Monetary Authority of Singapore (MAS); the City of London in the United Kingdom; Nice and Paris in France.

EDHEC-Risk has developed a close partnership with a small number of sponsors within the framework of research chairs or major research projects:

- *ETF and Passive Investment Strategies, in partnership with Amundi ETF*
- *Regulation and Institutional Investment, in partnership with AXA Investment Managers*
- *Asset-Liability Management and Institutional Investment Management, in partnership with BNP Paribas Investment Partners*
- *New Frontiers in Risk Assessment and Performance Reporting, in partnership with CACEIS*
- *Exploring the Commodity Futures Risk Premium: Implications for Asset Allocation and Regulation, in partnership with CME Group*

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- Asset-Liability Management Techniques for Sovereign Wealth Fund Management, *in partnership with Deutsche Bank*
- The Benefits of Volatility Derivatives in Equity Portfolio Management, *in partnership with Eurex*
- Structured Products and Derivative Instruments, *sponsored by the French Banking Federation (FBF)*
- Optimising Bond Portfolios, *in partnership with the French Central Bank (BDF Gestion)*
- Risk Allocation Solutions, *in partnership with Lyxor Asset Management*
- Infrastructure Equity Investment Management and Benchmarking, *in partnership with Meridiam and Campbell Lutyens*
- Risk Allocation Framework for Goal-Driven Investing Strategies, *in partnership with Merrill Lynch Wealth Management*
- Investment and Governance Characteristics of Infrastructure Debt Investments, *in partnership with Natixis*
- Advanced Modelling for Alternative Investments, *in partnership with Société Générale Prime Services (Newedge)*
- Advanced Investment Solutions for Liability Hedging for Inflation Risk, *in partnership with Ontario Teachers' Pension Plan*
- Active Allocation to Smart Factor Indices, *in partnership with Rothschild & Cie*
- Solvency II, *in partnership with Russell Investments*

- Structured Equity Investment Strategies for Long-Term Asian Investors, *in partnership with Société Générale Corporate & Investment Banking*

The philosophy of the Institute is to validate its work by publication in international academic journals, as well as to make it available to the sector through its position papers, published studies, and global conferences.

To ensure the distribution of its research to the industry, EDHEC-Risk also provides professionals with access to its website, [www.edhec-risk.com](http://www.edhec-risk.com), which is entirely devoted to international risk and asset management research. The website, which has more than 70,000 regular visitors, is aimed at professionals who wish to benefit from EDHEC-Risk's analysis and expertise in the area of applied portfolio management research. Its quarterly newsletter is distributed to more than 200,000 readers.

### EDHEC-Risk Institute: Key Figures, 2014-2015

Number of permanent staff	48
Number of research associates & affiliate professors	36
Overall budget	€6,500,000
External financing	€7,025,695
Nbr of conference delegates	1,087
Nbr of participants at research seminars and executive education seminars	1,465

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## Research for Business

The Institute's activities have also given rise to executive education and research service offshoots. EDHEC-Risk's executive education programmes help investment professionals to upgrade their skills with advanced risk and asset management training across traditional and alternative classes. In partnership with CFA Institute, it has developed advanced seminars based on its research which are available to CFA charterholders and have been taking place since 2008 in New York, Singapore and London.

In 2012, EDHEC-Risk Institute signed two strategic partnership agreements with the Operations Research and Financial Engineering department of Princeton University to set up a joint research programme in the area of asset-liability management for institutions and individuals, and with Yale School of Management to set up joint certified executive training courses in North America and Europe in the area of risk and investment management.

As part of its policy of transferring know-how to the industry, in 2013 EDHEC-Risk Institute also set up ERI Scientific Beta. ERI Scientific Beta is an original initiative which aims to favour the adoption of the latest advances in smart beta design and implementation by the whole investment industry. Its academic origin provides the foundation for its strategy: offer, in the best economic conditions possible, the smart beta solutions that are most proven scientifically with full transparency in both the methods and the associated risks.

# EDHEC-Risk Institute Publications and Position Papers (2013-2016)



# EDHEC-Risk Institute Publications (2013–2016)

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## 2016

- Maeso, J.M., L. Martellini. Factor Investing and Risk Allocation: From Traditional to Alternative Risk Premia Harvesting (June).
- Amenc, N., F. Goltz, V. Le Sourd, A. Lodh and S. Sivasubramanian. The EDHEC European ETF Survey 2015 (February).
- Martellini, L. Mass Customisation versus Mass Production in Investment Management (January).

## 2015

- Blanc-Brude, F., M. Hasan and T. Whittaker. Cash Flow Dynamics of Private Infrastructure Project Debt (November).
- Amenc, N., G. Coqueret, and L. Martellini. Active Allocation to Smart Factor Indices (July).
- Martellini, L., and V. Milhau. Factor Investing: A Welfare Improving New Investment Paradigm or Yet Another Marketing Fad? (July).
- Goltz, F., and V. Le Sourd. Investor Interest in and Requirements for Smart Beta ETFs (April).
- Amenc, N., F. Goltz, V. Le Sourd and A. Lodh. Alternative Equity Beta Investing: A Survey (March).
- Amenc, N., K. Gautam, F. Goltz, N. Gonzalez, and J.P Schade. Accounting for Geographic Exposure in Performance and Risk Reporting for Equity Portfolios (March).
- Amenc, N., F. Ducoulombier, F. Goltz, V. Le Sourd, A. Lodh and E. Shirbini. The EDHEC European Survey 2014 (March).
- Deguest, R., L. Martellini, V. Milhau, A. Suri and H. Wang. Introducing a Comprehensive Risk Allocation Framework for Goals-Based Wealth Management (March).
- Blanc-Brude, F., and M. Hasan. The Valuation of Privately-Held Infrastructure Equity Investments (January).

## 2014

- Coqueret, G., R. Deguest, L. Martellini, and V. Milhau. Equity Portfolios with Improved Liability-Hedging Benefits (December).
- Blanc-Brude, F., and D. Makovsek. How Much Construction Risk do Sponsors take in Project Finance. (August).
- Loh, L., and S. Stoyanov. The Impact of Risk Controls and Strategy-Specific Risk Diversification on Extreme Risk (August).
- Blanc-Brude, F., and F. Ducoulombier. Superannuation v2.0 (July).
- Loh, L., and S. Stoyanov. Tail Risk of Smart Beta Portfolios: An Extreme Value Theory Approach (July).



## EDHEC-Risk Institute Publications (2013–2016)

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- Foulquier, P. M. Arouri and A. Le Maistre. P. A Proposal for an Interest Rate Dampener for Solvency II to Manage Pro-Cyclical Effects and Improve Asset-Liability Management (June).
- Amenc, N., R. Deguest, F. Goltz, A. Lodh, L. Martellini and E.Schirbini. Risk Allocation, Factor Investing and Smart Beta: Reconciling Innovations in Equity Portfolio Construction (June).
- Martellini, L., V. Milhau and A. Tarelli. Towards Conditional Risk Parity – Improving Risk Budgeting Techniques in Changing Economic Environments (April).
- Amenc, N., and F. Ducoulombier. Index Transparency – A Survey of European Investors Perceptions, Needs and Expectations (March).
- Ducoulombier, F., F. Goltz, V. Le Sourd, and A. Lodh. The EDHEC European ETF Survey 2013 (March).
- Badaoui, S., Deguest, R., L. Martellini and V. Milhau. Dynamic Liability-Driven Investing Strategies: The Emergence of a New Investment Paradigm for Pension Funds? (February).
- Deguest, R., and L. Martellini. Improved Risk Reporting with Factor-Based Diversification Measures (February).
- Loh, L., and S. Stoyanov. Tail Risk of Equity Market Indices: An Extreme Value Theory Approach (February).

### 2013

- Lixia, L., and S. Stoyanov. Tail Risk of Asian Markets: An Extreme Value Theory Approach (August).
- Goltz, F., L. Martellini, and S. Stoyanov. Analysing statistical robustness of cross-sectional volatility. (August).
- Lixia, L., L. Martellini, and S. Stoyanov. The local volatility factor for asian stock markets. (August).
- Martellini, L., and V. Milhau. Analysing and decomposing the sources of added-value of corporate bonds within institutional investors' portfolios (August).
- Deguest, R., L. Martellini, and A. Meucci. Risk parity and beyond - From asset allocation to risk allocation decisions (June).
- Blanc-Brude, F., Cocquemas, F., Georgieva, A. Investment Solutions for East Asia's Pension Savings - Financing lifecycle deficits today and tomorrow (May)
- Blanc-Brude, F. and O.R.H. Ismail. Who is afraid of construction risk? (March)
- Lixia, L., L. Martellini, and S. Stoyanov. The relevance of country- and sector-specific model-free volatility indicators (March).
- Calamia, A., L. Deville, and F. Riva. Liquidity in European equity ETFs: What really matters? (March).

## EDHEC-Risk Institute Publications (2013–2016)

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- Deguest, R., L. Martellini, and V. Milhau. The benefits of sovereign, municipal and corporate inflation-linked bonds in long-term investment decisions (February).
- Deguest, R., L. Martellini, and V. Milhau. Hedging versus insurance: Long-horizon investing with short-term constraints (February).
- Amenc, N., F. Goltz, N. Gonzalez, N. Shah, E. Shirbini and N. Tessaromatis. The EDHEC european ETF survey 2012 (February).
- Padmanaban, N., M. Mukai, L. Tang, and V. Le Sourd. Assessing the quality of asian stock market indices (February).
- Goltz, F., V. Le Sourd, M. Mukai, and F. Rachidy. Reactions to "A review of corporate bond indices: Construction principles, return heterogeneity, and fluctuations in risk exposures" (January).
- Joenväärä, J., and R. Kosowski. An analysis of the convergence between mainstream and alternative asset management (January).
- Cocquemas, F. Towards better consideration of pension liabilities in European Union countries (January).
- Blanc-Brude, F. Towards efficient benchmarks for infrastructure equity investments (January).

# EDHEC-Risk Institute Position Papers (2013–2016)

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## 2016

- O'Kane.D. Initial Margin for Non-Centrally Cleared OTC Derivatives (June).

## 2014

- Blanc-Brude, F. Benchmarking Long-Term Investment in Infrastructure: Objectives, Roadmap and Recent Progress (June).

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